

Stochastic Model of Demand for Medical Care with Endogenous Labour Supply and Health Insurance

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Abstract

The paper proposes a model of demand for medical care under uncertainty. Both health capital and wealth are modelled as Wiener processes. The model uses a continuous time stochastic optimisation technique to derive optimal solutions for consumption, leisure and medical care. Insurance against uncertain medical expenditure is then incorporated into the optimisation problem under the assumption of constant relative risk aversion of the value function, and constant elasticity, relative risk aversion and relative prudence of the health investment function. The optimal solution is shown to depend on the curvature of the value function, the curvature of the health investment technology, and variances of the stochastic shocks. Dynamic simulations of the model are carried out.

Keywords: Demand for health; Uncertainty; Insurance; Stochastic optimisation

JEL Classification: C61, D81, I12

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Introduction

It has been recognised in the mainstream literature on health capital, pioneered by Michael Grossman in Grossman (1972), that medical care should be viewed as one of the inputs into the health capital production function. It is the commodity of “good health” that individuals are demanding, not medical care per se. A number of studies have been devoted to modelling the demand for health using generalised Grossman models, including Muurinen (1982), Ehrlich and Chuma (1990), Reid (1998), and Grossman (1999). These models are deterministic dynamic optimisation models, where the terminal period occurs once the level of health capital falls below some predetermined critical level. An important feature of these analyses is the time input into production of health. An agent derives utility from the consumption good, and disutility from sick time, while the rest of his time is allocated for market activities and investment into health and household production of a consumption good.

Since Arrow's (1963) paper on uncertainty and economics of medical care, it has been recognised that predictions of the stochastic model might be quite different from those of the deterministic one. An important extension of the model of demand for health capital has attempted to address the random nature of health, illness and death. Stochastic models of this type were developed in Cropper (1977), Dardanoni and Wagstaff (1990), and Picone *et al.* (1998). In Cropper (1977), the critical value of health stock is assumed to be a random variable drawn from a specific distribution. Picone *et al.* (1998) use a dynamic Grossman model with uncertainty entering the health capital accumulation equation multiplicatively in the level of

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medical expenditure. Their model does not allow for the closed form solution and is implemented numerically. None of those studies model insurance against uncertain medical expenditure, or health insurance.

Studies of health insurance comprise an important part of the health economics literature and include Spence and Zeckhauser (1971), Ehrlich and Becker (1972), Feldstein (1973), Friedman (1974), and the RAND Health Insurance Study, whose results are reported in Newhouse and The Insurance Experiment Group (1993) and in a number of individual papers. The coinsurance and other price elasticities of the demand for medical care were studied in Phelps (1973), Phelps and Newhouse (1974), van de Ven and van Praag (1981). It has been established that demand for medical services is higher at a lower coinsurance rate or a lower deductible, and that the optimal spending strategy under the latter policy option exhibits a non-linear behavior.

There are a number of problems associated with the provision of health insurance - agency costs, moral hazard, adverse selection and supplier-induced demand to name just a few. The health status of the insured is not directly observable to the insurance company, thus the insurance policy under such asymmetric information will necessarily be second-best. The adverse selection leads to the separating equilibrium in the insurance markets, with different health groups choosing different levels of coverage¹. The moral hazard arises when insurance holders demand more medical care than uninsured or insured under the less generous plans. A number of studies including Feldstein (1973), Feldman and Dowd (1991), and Manning and Marquis (1996), evaluated a welfare loss associated with moral hazard in the health insurance market, and were aiming to construct an optimal insurance policy that balances benefits of risk spreading with losses attributed to the moral hazard.

The models of medical insurance have been developed in several studies including Zeckhauser (1970), Phelps (1973), Marquis and Holmer (1996), and Blomqvist (1997). Zeckhauser (1970) studied how the optimal choice of the sharing function for the insured, subject to the varying ability of the insurance company to discriminate against different types of insureds according to

their health status, would alter the premiums charged for the plan. Van de Ven and van Praag (1981) assumed a lognormal distribution for the health expenditure and used an adjusted Tobin model to study the effect of insurance on the demand for health services. Blomqvist (1997) used a dynamic optimisation technique to construct an optimal non-linear health insurance contract with an exogenous income. A recent paper by Liljas (1998) develops a stochastic model of the demand for health that incorporates insurance against loss of income due to illness. The model was further improved by Tabata and Ohkusa (2000).

This paper develops a dynamic stochastic model of demand for medical care and health insurance under the assumption of lognormality of underlying wealth and health distributions. The model incorporates a correlation between health and wealth processes without making health a direct function of income, which was one of the assumptions in Contoyannis and Forster (1999). It is a continuous time model that rests on a probabilistic assumption of lognormality of health justified by the previous studies including Wagstaff and van Doorslaer (1994) and Gerdtham, Johannesson et al. (1999).

There are several departures in this paper from Grossman's original framework. First, there is no time input into the health production function. Second, the consumer's utility depends on the health-adjusted leisure as opposed to the "healthy time". This allows us to model labour supply explicitly and to endogenise income. Life span is not endogenous in this paper: the representative consumer is alive as long as her health index is positive. Future extension of the model could incorporate an endogenous time of death by defining a positive threshold for the health capital below which death occurs.

The structure of the paper is as follows: Section 2 contains a brief discussion of the stochastic optimisation technique used in the paper. Section 3 proposes a stochastic model of demand for medical care under the aforementioned assumptions, compares results with a certainty case and illustrates them for logarithmic utility function. Section 4 extends the model by introducing

¹ See Rothschild and Stiglitz (1976) for the discussion of the existence of equilibrium and its structure with imperfect information. The equilibrium with the adverse selection in the health insurance market is discussed in Cutler and Zeckhauser (1999).

health insurance and finds that the distribution of optimal medical expenditure changes after the introduction of insurance. Results of the analysis are summarised in the Conclusions.

2. Stochastic Control Problem and Itô's Lemma

The stochastic optimization technique² used in solving the model of the next sections can be summarised as follows. If the stochastic process dy is generated by the equation

$$dy = f(y, t)dt + dv, \quad (2.1)$$

where $dv \sim N(0, \Sigma(y, t)dt)$, then for any twice differentiable in its arguments function $G(y, t)$ its stochastic differential is given by the Itô's Lemma:

$$\begin{aligned} dG &= G(y(t+dt), t+dt) - G(y(t), t) = \frac{\partial G}{\partial t} dt + \left(\frac{\partial G}{\partial y}\right)' dy + \frac{1}{2} (dy)' \frac{\partial^2 G}{\partial y \partial y'} dy + o(dt) = \\ &= \left[\frac{\partial G}{\partial t} + \left(\frac{\partial G}{\partial y}\right)' f + \frac{1}{2} tr(G_{yy} \Sigma) \right] dt + \left(\frac{\partial G}{\partial y}\right)' dv \end{aligned} \quad (2.2)$$

where tr stands for the trace of a matrix, and stochastic process dy is defined in (2.1).

A differential generator $L_y[G(y, t)]$ of the function $G(y, t)$ is defined as expected rate of change in

$$G(y(t), t), \text{ when evolution of } dy \text{ is given by (2.1), that is, } L_y[G(y, t)] = \lim_{dt \rightarrow 0} E_t \left[\frac{dG}{dt} \right].$$

As it follows from (3.2), the differential generator of $G(y, t)$ with dy given in (2.1) equals

$$L_y[G(y, t)] = \frac{\partial G}{\partial t} + \left(\frac{\partial G}{\partial y}\right)' f + \frac{1}{2} tr(G_{yy} \Sigma). \quad (2.3)$$

The stochastic control problem is generally stated as finding

$$\max_{x_s} \tilde{V}(y(0), 0) = E_0 \int_0^{\infty} U(y(s), x(s), s) ds,$$

² See Turnovsky (1995) Chapter 14, Malliaris and Brock (1982) Chapter 2.

subject to stochastic accumulation equation (2.1) for the state variable y_s and x_s being control variable. The optimum solution has to satisfy the stochastic Bellman equation

$$0 = \max_{x_s} \{U(y(t), x(t), t) + L_y[\tilde{V}(y(t), t)]\}, \quad (2.4)$$

where the control variables x_s are chosen so that they satisfy their first-order optimality

conditions, and $\tilde{V}(y(t), t)$ is a value function defined by $\tilde{V}(y(t), t) = \max_{x_s} E_t \int_t^{\infty} U(y(s), x(s), s) ds$.

3. Stochastic Model Of Demand For Medical Care

Consider a stochastic version of the representative agent model. The model is formulated in real terms, and the consumer is optimizing expected utility of the stream of consumption and health-adjusted leisure, with the adjustment factor given by $f(H_t)$, where $f' > 0$, $f'' < 0$, and H_t is the current health status:

$$\max_{c_t, I_t, m_t} E_0 \int_0^{\infty} U(c_t, f(H_t)I_t) e^{-rt} dt, \quad (3.1)$$

with respect to consumption c_t , leisure I_t , and medical expenditure m_t , subject to the dynamic constraints discussed below.

The health capital H_t is assumed to be governed by the stochastic accumulation equation, with the variance proportionate to the level of health capital. The depreciation rate is given by d_t , and the investment into health capital stock on the interval of the length dt is given by the expression $y(m_t)H_t dt$, with $y' > 0$, $y'' < 0$, $y''' > 0$. The stochastic component is assumed to be a Wiener process, $dh_t \sim N(0, \sigma_H^2 dt)$. The evolution of health capital is given by

$$dH_t = (y(m_t) - d_t)H_t dt + H_t dh_t. \quad (3.2)$$

Analogously, change in wealth is compounded from the interest earned on the stock of wealth over time dt , income from market activities, less expenditure on consumption stream and medical care. The unit of labour supplied by the consumer is assumed to be paid an efficiency wage, dependent on her health status. This would reflect changes in productivity of labour due to changes in consumer's health. The efficiency coefficient is given by the function $e(H_t)$, $e' > 0$, $e'' < 0$. The error term enters multiplicatively in W_t , in line with the health accumulation equation. The real price of medical services in terms of consumption good is given by c_t . The stochastic wealth accumulation equation is given by

$$dW_t = (r_t W_t + w_t e(H_t)(1 - I_t) - c_t - c_t m_t) dt + W_t dw_t, \quad (3.3)$$

where w_t is a wage rate, r_t is a real interest rate, and $dw_t \sim N(0, s_W^2 dt)$ is a Wiener process for the wealth disturbances. The instantaneous covariance between the two disturbances is given by $\text{cov}(dh_t, dw_t) = s_{HW} dt$. The initial values for stocks of health and wealth are given by H_0 and W_0 respectively. A consumer maximises expected utility (3.1) subject to two stochastic accumulation equations (3.2), (3.3). This problem represents a case of a self-insured consumer who spreads her own risks by purchasing medical care and smoothing consumption and leisure under uncertainty.

Applying the stochastic optimisation technique discussed in Section 2 to the problem (3.1) - (3.3), note that $y_t = (H_t, W_t)'$, $x_t = (c_t, I_t, m_t)'$, and

$$\begin{aligned} L_y[\tilde{V}(y(t), t)] = & \frac{\partial \tilde{V}}{\partial t} + \frac{\partial \tilde{V}}{\partial H} (y(m_t) - d_t) H_t + \frac{\partial \tilde{V}}{\partial W} (r_t W_t + (1 - I_t) w e(H_t) - c_t - c_t m_t) + \\ & + \frac{1}{2} \frac{\partial^2 \tilde{V}}{\partial H^2} H^2 s_H^2 + \frac{\partial^2 \tilde{V}}{\partial H \partial W} H W s_{HW} + \frac{1}{2} \frac{\partial^2 \tilde{V}}{\partial W^2} W^2 s_W^2 \end{aligned}$$

To simplify notations, the time index is dropped, and subscripts will denote partial derivatives with respect to the relevant variable. The value function is assumed to be of the form similar to

the integrand in the utility functional: $\tilde{V}(H, W; t) = V(H, W, t)e^{-rt}$.

$$\text{Consider } H = e^{-rt}U(c, f(H)I) + L_y[\tilde{V}(H, W; t)] = \quad (3.4)$$

$$\begin{aligned} &= e^{-rt}U(c, f(H)I) + V_t e^{-rt} - r\tilde{V} + \tilde{V}_H(y(m) - d)H + \tilde{V}_W(rW + we(H)(1 - I) - c - cm) + \\ &+ \frac{1}{2}\tilde{V}_{HH}H^2s_H^2 + \tilde{V}_{HW}HWs_{HW} + \frac{1}{2}\tilde{V}_{WW}W^2s_W^2 \end{aligned}$$

Maximising (3.4), the first order necessary conditions for the optimal choice of controls are

$$\frac{\partial H}{\partial c} = 0, \frac{\partial H}{\partial I} = 0, \frac{\partial H}{\partial m} = 0, \text{ which implies that}$$

$$U_c = V_W, \quad (3.5.a)$$

$$U_I f(H) = wV_W e(H), \quad (3.5.b)$$

$$V_{HY}'(m)H = cV_W \quad (3.5.c)$$

Expression (3.5.a) is a standard condition equating the marginal cost of reducing current period consumption by one unit to the marginal benefits of having an extra unit of wealth available. Condition (3.5.b) states that the health-adjusted utility gain from the additional unit of leisure is equal at the optimum to the efficiency-adjusted loss of an extra unit of time spent on wage-earning activities. The third condition (3.5.c) states that the gain in terms of the value of health of the marginal product of the unit of medical expenditure in the production of new health must be equal to the marginal loss of c units of wealth, where c is a price per unit of medical care. These are standard and intuitive interpretations of the FONCs (3.5).

In addition, the Bellman equation has to be satisfied when the optimal values from the FONCs (3.5) are substituted, which in this case is equivalent to

$$0 = \max_{c, I, m} \{U(c, f(H)I) - rV + V_t + V_H(y(m) - d)H + V_W(rW + we(H)(1 - I) - c - cm) + \frac{1}{2}V_{HH}H^2s_H^2 + V_{HW}Hws_{HW} + \frac{1}{2}V_{WW}W^2s_W^2\} \quad (3.6)$$

The solution of the FONCs (3.5.a-c) are functions of H , W , and t , $c = c(H, W, t)$, $I = I(H, W, t)$, $m = m(H, W, t)$, and the partial derivatives of the value function are of the form $V_H = V_H(H, W, t)$, $V_W = V_W(H, W, t)$. Partially differentiating the Bellman equation (3.6) with respect to H and W , and using the first order optimality conditions (3.5.a-c) along with Itô's lemma, one could establish that under the additional assumption of $\mathbf{h}_f = -H \frac{f'(H)}{f(H)}$ and $\mathbf{h}_e = -H \frac{e'(H)}{e(H)}$, $\mathbf{h}_f = \mathbf{h}_e \stackrel{def}{=} \mathbf{h} < 0$ - constant elasticity of $f(\cdot)$, $e(\cdot)$ with respect to H , the following dynamic stochastic equations for marginal value of health and wealth can be obtained (see Appendix A for derivation):

$$dV_H = \left\{ \left[r + d - y(m) \left(1 - \frac{we(H)y'(m)h}{cy(m)} \right) \right] V_H - V_{HH}Hs_H^2 - V_{HW}Wws_{HW} \right\} dt + V_{HH}Hdh_t + V_{HW}Wdw_t \quad (3.7)$$

$$dV_W = \left\{ (r - r)V_W - V_{HW}Hws_{HW} - V_{WW}Ws_W^2 \right\} dt + V_{WH}Hdh_t + V_{WW}Wdw_t. \quad (3.8)$$

Equations (3.5.a-c), (3.7) and (3.8) constitute an equilibrium solution for the stochastic optimisation problem (3.1)-(3.3).

3.1. Example: No Uncertainty

The general solution of the previous section could be applied in the absence of risk as well.

Assuming that $dw_t = dh_t = 0$, and $s_W^2 = s_H^2 = s_{HW} = 0$, one could get the FONCs (3.5.a-c) and the following deterministic evolution for marginal values of wealth and health:

$$dV_W = (r - r)V_W, \quad dV_H = (r + d - y(m) \left[1 - \frac{w e(H) y'(m) h}{y(m) c} \right]) V_H, \text{ from which the familiar Euler's}$$

equation follows: $\frac{1}{U_c} \frac{dU_c}{dt} = r - r$.

The shadow price of wealth, or marginal impact of wealth on the value function, depends on the interest rate and a subjective discount rate only. A much larger set of model parameters determine the shape of marginal value of health schedule. Optimal marginal valuation of health is positively related to the subjective discount rate, the depreciation rate of health capital, price of medical care, increases in which lead to the lower optimal stock of health capital. Marginal value of health at optimum is lowered by increases in the health investment rate, wage rate, efficiency parameter and the steepness of the health investment schedule, all of which implies higher optimal health capital stock.

Note that marginal valuation of health and wealth under uncertainty differs from the certainty case: under regular assumptions about the value function, an increase in variance of health process leads to a higher marginal value of health (and lower equilibrium stock of health). The positive correlation between wealth and health shocks has a negative effect on the expected marginal value of health (hence, a positive effect on the equilibrium level of health capital), and a negative effect on the expected marginal value of wealth (positive effect on the equilibrium stock of wealth). Higher uncertainty about wealth translates into higher expected marginal value of wealth and is associated with the lower optimal level of wealth. Variance of wealth (health) adjusted by the risk aversion parameter of the value function serves as an extra discount factor in the value of wealth (health) accumulation equation.

3.2. Example: Logarithmic Utility Function with Uncertainty

Let us assume that the utility function is given by

$$U(c, f(H)I) = a \ln c + b \ln[f(H)I]. \quad (3.9)$$

The health-adjustment function and the efficiency function are of the constant elasticity form

with $-\mathbf{h}_f = -\mathbf{h}_e \stackrel{def}{=} \mathbf{h}$, that is, they are $f(H) = a_f H^{\mathbf{h}}$ and $e(H) = a_e H^{\mathbf{h}}$ respectively, where $a_f, a_e, \mathbf{h} \in \mathbb{R}^+$. Assume that the investment function is logarithmic: $y(m) = \mathbf{y} \ln m$, $\mathbf{y} > 0$.

Given the logarithmic form of the utility function, we will be looking for a similar, logarithmic value function for the problem (3.1) -(3.3), $\tilde{V}(H, W; t) = V(H, W, t)e^{-rt}$, where $V(H, W, t) = a_0 \ln W + b_0 \ln H + g_0$. (3.10)

The first-order conditions (3.5.a-c) imply that the optimal solutions for the choice variables are

$$c = \frac{a}{a_0} W, \quad I = \frac{b}{a_0 w a_e H^{\mathbf{h}}} W, \quad \text{and} \quad m = \frac{b_0 \mathbf{y}}{a_0 c} W. \quad (3.11)$$

The Bellman equation has to be satisfied identically at the optimum, which makes it possible to solve for the unknown parameters a_0, b_0, g_0 in (3.10). Performing calculations presented in Appendix B, the value function is given by the following expression:

$$V(H, W, t) = \frac{a + b}{r + a_e w (1 - \frac{h\mathbf{y}}{r})} \ln W + \frac{w a_e (a + b)}{r + a_e w (1 - \frac{h\mathbf{y}}{r})} \frac{h}{r} \ln H + g_0 \quad (3.12)$$

The optimal consumption, according to the FONCs, is given by

Assuming that $\frac{h\mathbf{y}}{r} < 1$, the optimal consumption, according to the FONCs, is given by

$$c = \frac{a}{a + b} (r + a_e w (1 - \frac{h\mathbf{y}}{r})) W, \quad \text{the leisure} \quad I = \frac{b}{a + b} \frac{r + a_e w (1 - \frac{h\mathbf{y}}{r})}{w a_e H^{\mathbf{h}}} W, \quad \text{and the optimal}$$

medical expenditure is given by $m = \frac{hywa_e}{rc} W$.

Note that medical care demanded does not depend on the contemporaneous health status, which reflects the smoothing effect of the optimal solution. Another result to notice is that the consumption-to-leisure ratio along the optimal path equals $\frac{c}{l} = \frac{awe(H)}{b}$. It is determined by the underlying parameters of the utility function, wage rate and the current health status which acts through the efficiency adjustment function.

The solution of this particular case has been illustrated by running simulations of the Wiener processes for disturbances under the assumption of zero correlation between them. Constant real interest rate, wage, health capital depreciation rate and price of medical care were assumed throughout the simulations. For a particular realization of the random disturbances, different scenarios concerning model parameters and initial values were investigated.

The paths of wealth and health capital, their marginal values, and optimal solutions for consumption, leisure and medical expenditure when parameter a (a relative weight of consumption in the utility function) varies, are presented in Figure 1. Higher weight put on consumption quite intuitively leads to higher optimal consumption, lower optimal leisure (hence, higher labour supply) and to lower optimal medical spending, which results in a lower optimal level of health capital and lower efficiency of labour. The marginal valuation of wealth and optimal stock of wealth are not affected.

4. Stochastic Model Of Demand For Medical Care With Insurance

The purpose of this section is to introduce insurance into the stochastic model developed in the previous sections of the paper. The insurance company observes the distribution of the optimal medical expenditure of the self-insured consumer and assumes this distribution will not change after introduction of insurance (no moral hazard assumption). The consumer will be liable for

part of her medical spending, that is, the insurance contract includes a positive coinsurance parameter, k . An insurance premium is paid at time t , and medical spending is incurred over the interval $(t, t+dt)$, of which a fraction k is paid by the insured. The question of how the insurance company is financed is not addressed explicitly.

First, it is necessary to establish the equation for the evolution of the medical spending. With the stochastic equation for dm_t in hand, the expected amount of medical care consumed over the interval $(t, t+dt)$ would be given by $E_t dm_t$, and the actuarially fair premium flow charged by the insurance company at time t would be given by $p_t = (1-k)c \lim_{dt \rightarrow 0} E_t \frac{dm_t}{dt}$ under the assumption of zero loading.

4.1. Stochastic accumulation equation for demand for medical services

As shown in Appendix C, under the additional assumptions of constant relative risk aversion of the utility and value functions, and constant elasticity, relative risk aversion and relative prudence of the health investment schedule, the distribution of optimal medical expenditure of the selfinsured consumer of Section 3 of this paper is given by

$$dm = \left[\frac{m}{h_2} \left\{ r - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right] dt + dm_t, \quad (4.1)$$

$$\text{where } -\frac{my'(m)}{y(m)} = h_1, \quad -\frac{my''(m)}{y'(m)} = h_2, \quad -\frac{my'''(m)}{y''(m)} = h_3, \quad (4.2)$$

$h_1 < 0$, $h_j \in R^+$, $j = 2, 3$, and $n_j, j = 1, 3$ are constants depending on the curvature of the value function and on the variances and covariance of the disturbance processes.

Based on the observed distribution of medical expenditure by a selfinsured consumer (4.1), the

insurance company sets the instantaneous premium equal to

$$p = (1-k)c \left[\frac{m}{h_2} \left\{ r - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right]. \quad (4.3)$$

4.2. Optimisation Problem With The Insurance Contract

The consumer is optimising the expected utility of the stream of consumption and health-

$$\text{adjusted leisure: } \max_{c_t, I_t, m_t} E_0 \int_0^{\infty} U(c_t, f(H_t)I_t) e^{-rt} dt, \quad (4.4)$$

with respect to consumption c_t , leisure I_t , and medical expenditure m_t , subject to the dynamic constraints discussed below. The health -adjustment function $f(H_t)$ is assumed to be $f' > 0$, $f'' < 0$, and $h_f = h$.

The health capital is assumed to be governed by the following stochastic accumulation equation:

$$dH_t = (y(m_t) - d_t)H_t dt + H_t dh_t, \quad (4.5)$$

where the investment function $y(m_t)$ is assumed to satisfy conditions (4.2) and $dh_t \sim N(0, s_H^2 dt)$.

In view of the discussion about the nature of the insurance contract and the amount of premium charged, the wealth accumulation equation is as follows:

$$\begin{aligned} dW_t &= (r_t W_t + w_t e(H_t)(1 - I_t) - c_t - p_t - k c_t m_t) dt + W_t dw_t = \\ &= (r_t W_t + w_t e(H_t)(1 - I_t) - c_t - \\ &\quad - (1-k)c_t \left[\frac{m_t}{h_2} \left\{ r_t - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y(m_t) \left\{ \frac{w_t}{c_t} e(H_t)h - n_2 \right\} \right] - k c_t m_t) dt + W_t dw_t, \end{aligned} \quad (4.6)$$

where w_t is a wage rate, r_t is a real interest rate, c_t is a real price of medical services in

terms of consumption good, and $dw_t \sim N(0, s_W^2 dt)$ is a Wiener process for the wealth disturbances. The instantaneous covariance between the two disturbances is given by $\text{cov}(dh_t, dw_t) = s_{HW} dt$. The efficiency function $e(H_t)$ satisfies the conditions $e' > 0$, $e'' < 0$, and $\mathbf{h}_e = \mathbf{h}$. The initial values for stocks of health and wealth are given by H_0 and W_0 respectively.

A consumer maximises expected utility (4.4) subject to two stochastic accumulation equations (4.5), (4.6). Applying the familiar stochastic optimisation technique, form the value function $\tilde{V}(H, W; t) = V(H, W, t)e^{-rt}$. As previously, the time index is dropped, and subscripts denote partial derivatives. Consider

$$H = e^{-rt} U(c, f(H)I) + L_y[\tilde{V}(H, W; t)] = \quad (4.7)$$

$$\begin{aligned} &= e^{-rt} U(c, f(H)I) + V_t e^{-rt} - r\tilde{V} + \tilde{V}_H (y(m) - \mathbf{d})H + \tilde{V}_W (rW + \mathbf{w}e(H)(1 - I) - c - kcm - \\ &- (1 - k)c \left[\frac{m}{h_2} \left\{ r - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y'(m) \left\{ \frac{w}{c} e(H)\mathbf{h} - n_2 \right\} \right]) + \\ &+ \frac{1}{2} \tilde{V}_{HH} H^2 s_H^2 + \tilde{V}_{HW} H W s_{HW} + \frac{1}{2} \tilde{V}_{WW} W^2 s_W^2. \end{aligned}$$

Maximising (4.7), the first order necessary conditions for the optimal choice of controls are:

$$U_c = V_W, \quad (4.8.a)$$

$$U_I f(H) = w V_W e(H), \quad (4.8.b)$$

$$V_H y'(m) H = c V_W \left\{ (1 - k) \left[\frac{r - n_1 + \frac{h_3}{h_2} n_3}{h_2} - \frac{h_1}{h_2} y'(m) \left\{ \frac{w}{c} e(H)\mathbf{h} - n_2 \right\} \right] + k \right\} \quad (4.8.c)$$

It is obvious that the first two FONCs are identical to the previous case without an insurance, while the optimality condition for the choice of m has changed.

Following the procedure described in the Appendix D, the following equations for evolution of the marginal values of health and wealth are derived:

$$dV_H = \left\{ [r + d - y(m)]V_H + \frac{V_W w e(H)h}{H} \left\{ 1 + \frac{h_1}{h_2} (1-k)y(m)h \right\} - \right. \\ \left. - V_{HH} H s_H^2 - V_{HW} W s_{HW} \right\} dt + V_{HH} H dh_t + V_{HW} W dw_t \quad (4.9)$$

$$\text{and } dV_W = \left\{ (r - r)V_W - V_{HW} H s_{HW} - V_{WW} W s_W^2 \right\} dt + V_{WH} H dh_t + V_{WW} W dw_t \quad (4.10)$$

The wealth accumulation equation (4.10) for the problem with insurance is identical to the equation (3.8) without any insurance. Re-writing the equation (4.8.c),

$$V_H y'(m) H = \\ = c V_W \left\{ (1-k) \left[\frac{r - n_1 + \frac{h_3 n_3}{h_2}}{h_2} - \frac{h_1}{h_2} y'(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right] + k \right\} \equiv c V_W \Leftrightarrow \\ \Leftrightarrow (1-k) \left[\frac{r - n_1 + \frac{h_3 n_3}{h_2}}{h_2} - \frac{h_1}{h_2} y'(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right] + k = 1 \Leftrightarrow \quad k=1.$$

When the coinsurance rate equals one, insurance does not technically exist, and the optimal solution for medical expenditure is equivalent to the self-insurance case covered in Section 3. The evolution of marginal value of health given by (4.9) when $k=1$ is identical to the previously derived (3.7), which could be re-written, using the FONCs, as

$$dV_H = \left\{ [r + d - y(m)]V_H + \frac{V_W w e(H)h}{H} - V_{HH} H s_H^2 - V_{HW} W s_{HW} \right\} dt + V_{HH} H dh_t + V_{HW} W dw_t.$$

It is not difficult to notice that marginal valuation of health with insurance differs from the case without insurance by the term $\frac{h_1}{h_2} (1-k)y(m) \frac{V_W w e(H)h^2}{H}$ which is negative ($h_1 < 0, h_2 > 0$).

Hence, in presence of insurance, marginal valuation of health is lower, and optimal health capital stock is higher than without insurance. For any $k \in (0,1)$, the initial assumption made by the insurance company of no moral hazard in setting their premiums leads to changes in the optimal solution on the consumer's part, and proves to be wrong. To study direction of change in optimal paths for consumption, leisure and medical expenditure, we need to introduce further restrictions on the random processes for disturbances and curvature of the value function. Even though the optimality condition for leisure looks the same with insurance as without, it depends on the stock of health that changes at optimum according to (4.9), so the path might change as well, inducing changes to consumption through the wealth accumulation equation.

4.3. Example: Logarithmic Utility Function

The problem with insurance was simulated under the assumptions of logarithmic utility function covered in Section 3.1.2, but the health investment schedule was assumed to be a CRRA form,

$$y(m) = \frac{z}{1-y} m^{1-y} .$$

For $y = 0.9$ and under the assumption of zero correlation between the

disturbance processes and constant real interest rate, wage, health capital depreciation rate and price of medical care, the impact of changes in coinsurance rate from one (no insurance) to 50% and 20% were simulated. Results are presented in Figure 2, which demonstrates that with a lower out-of-pocket price of medical care, the optimal medical expenditure rises, which leads to the higher health capital stock. The premium calculated on the basis of the pre-insurance distribution of medical expenditure is higher at a lower coinsurance rate. This Figure illustrates the fact that distribution of medical expenditure does change after the introduction of health insurance, and the assumption of no moral hazard in setting the premiums is not justified. Proper adjustment to the premium has to be made, which would take the moral hazard effect into account .

Conclusions

This paper contributes to the research into demand for health and medical care under uncertainty by constructing a continuous time stochastic model in which both health and wealth are governed by possibly correlated Wiener processes. The model includes endogenous leisure (labour supply) decision. The optimal solutions for consumption, leisure and medical expenditure are first derived in the case of a self-insured consumer. It is shown that the presence of uncertainty changes the marginal evaluation of both health and wealth compared to the deterministic case, with variances of the shock components adjusted by the parameters of risk aversion being additional discount factors. The model is then extended to incorporate health insurance under the assumption that the insurance company sets the premiums based on the distribution of medical expenditure observed for a selfinsured consumer. The results of the analysis show that the no moral hazard assumption proves to be wrong: after insurance is introduced, the optimal solution changes and the marginal evaluation of health decreases, hence the optimal stock of health increases. An increase in the optimal consumption of medical care after the introduction of insurance is illustrated by computer simulations.

Figure 1. Optimal wealth, health, V_W , V_H , efficiency, consumption, leisure and medical expenditure for various a .

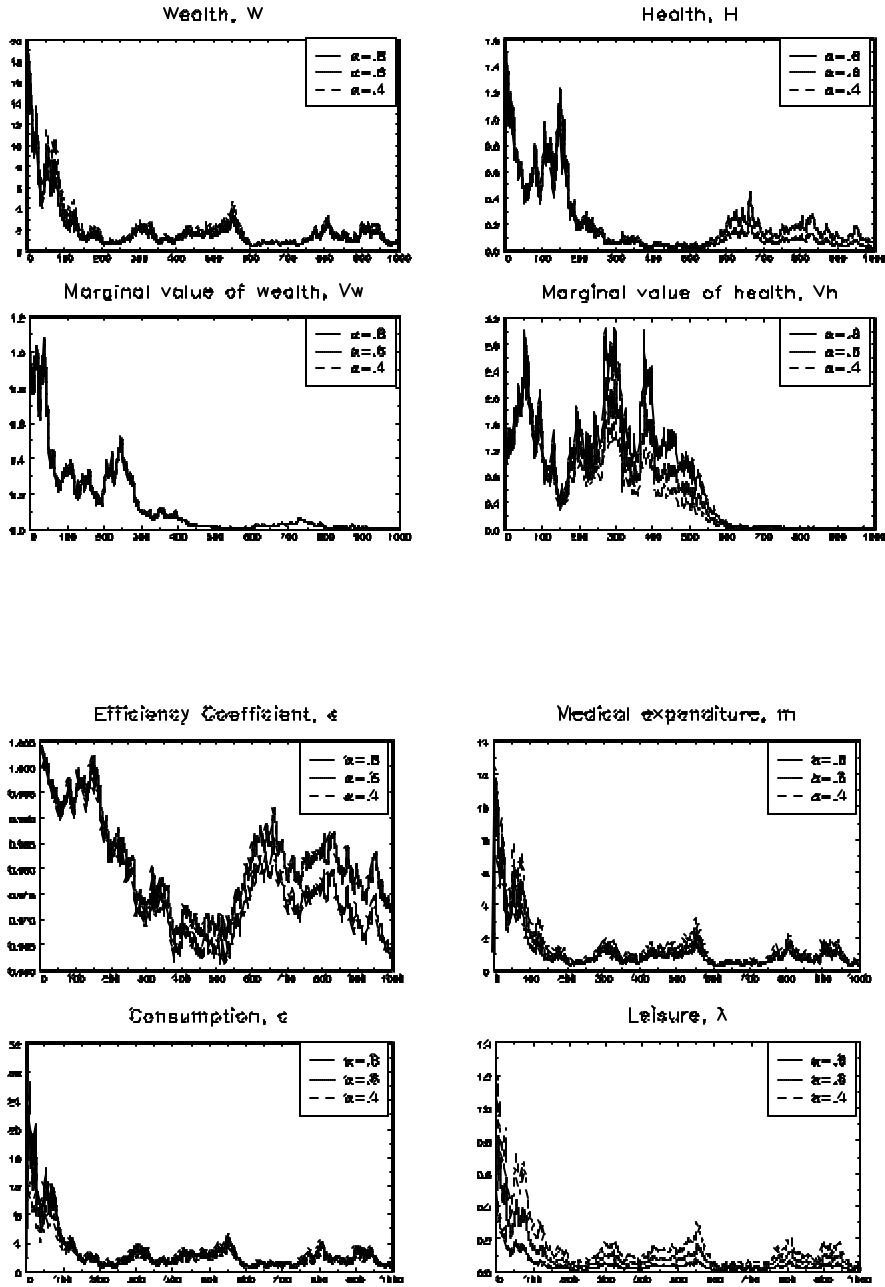
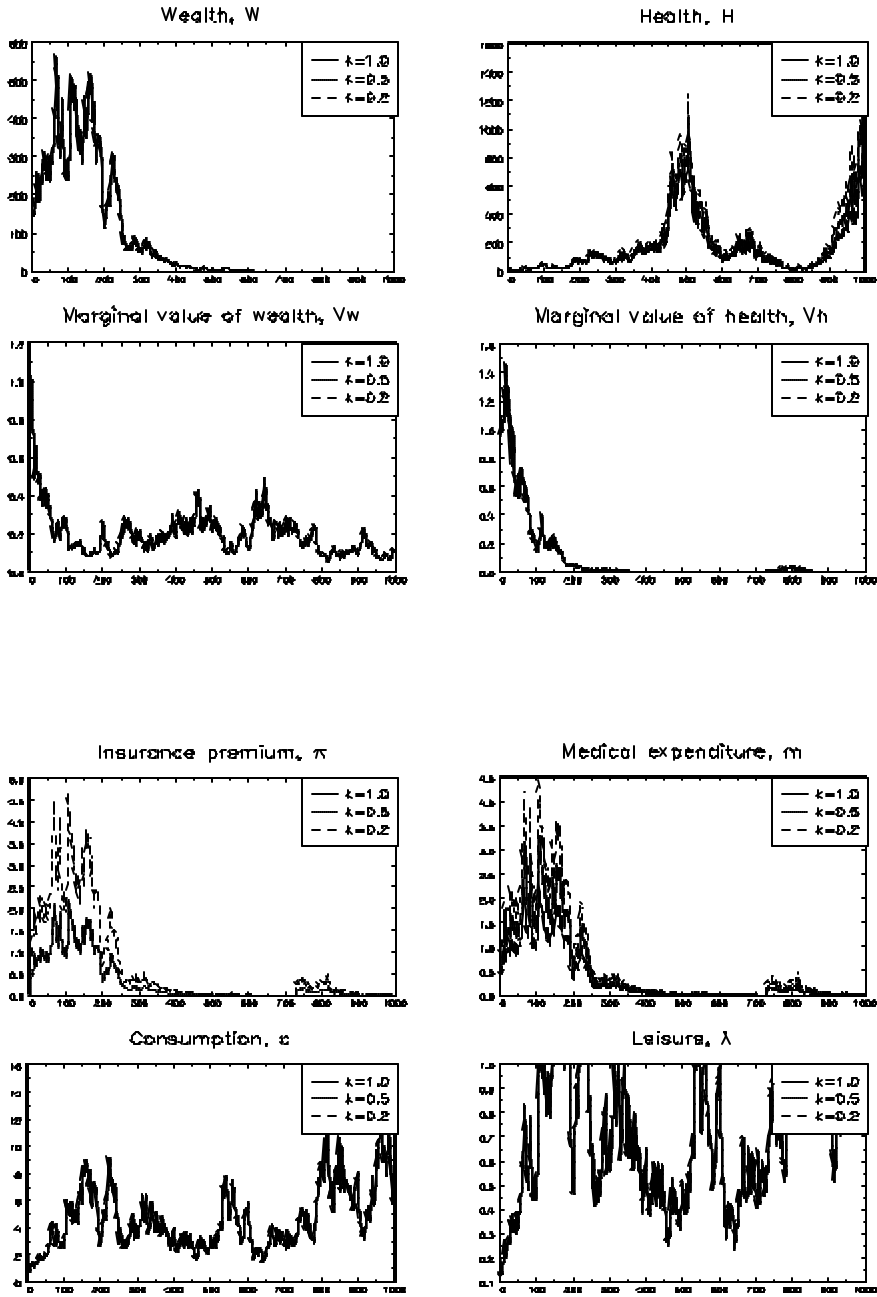


Figure 2. Optimal wealth, health, V_W , V_H , insurance premium, consumption, leisure and medical expenditure for various values of coinsurance parameter, k



Appendix A. Derivation of evolution of marginal value of health and wealth (no insurance)

Partially differentiating the Bellman equation (3.6) with respect to H , and using the first order optimality conditions (3.5.a-c), one could establish that

$$\begin{aligned}
 & (y(m) - r - d)V_H + U_1 f'(H)I + V_{Ht} + V_{HH}(y(m) - d)H + \\
 & + V_{WH}(rW + (1-I)we(H) - c - cm) + V_W we'(H)(1-I) + \\
 & + \frac{1}{2} V_{HHH} H^2 s_H^2 + V_{HHW} HW s_{HW} + \frac{1}{2} V_{HWW} W^2 s_W^2 + V_{HH} H s_H^2 + V_{HW} W s_{HW} = 0
 \end{aligned} \tag{A.1}$$

This could be further simplified by noting that $U_1 f'(H)I = \frac{V_W we(H) f'(H)}{f(H)} I$, and collecting

terms containing V_W in (A1) yields the following expression:

$$\begin{aligned}
 & (y(m) - r - d)V_H + V_{Ht} + V_{HH}(y(m) - d)H + \\
 & + V_{HW}(rW + (1-I)we(H) - c - cm) + V_W we \left(\frac{f'(H)}{f(H)} I + \frac{e'(H)}{e(H)} (1-I) \right) + \\
 & + \frac{1}{2} V_{HHH} H^2 s_H^2 + V_{HHW} HW s_{HW} + \frac{1}{2} V_{HWW} W^2 s_W^2 + V_{HH} H s_H^2 + V_{HW} W s_{HW} = 0
 \end{aligned} \tag{A.2}$$

Finally, by noting that $\frac{V_W}{H} = \frac{V_H y'(m)}{c}$, and by denoting $h_f = -H \frac{f'(H)}{f(H)}$ and $h_e = -H \frac{e'(H)}{e(H)}$,

$h_f, h_e < 0$, equation (A.2) is reduced to the following:

$$\begin{aligned}
 & \left[y(m) - r - d - \frac{we(H)y'(m)}{c} (h_f I + h_e (1-I)) \right] V_H + V_{Ht} + V_{HH}(h(m) - d)H + \\
 & + V_{WW}(rW + (1-I)we(H) - c - cm) + \\
 & + \frac{1}{2} V_{HHH} H^2 s_H^2 + V_{HHW} HW s_{HW} + \frac{1}{2} V_{HWW} W^2 s_W^2 + V_{HH} H s_H^2 + V_{HW} W s_{HW} = 0
 \end{aligned} \tag{A.3}$$

Using Itô's Lemma,

$$dV_H = V_{Ht} dt + V_{HH} dH + V_{HW} dW + \frac{1}{2} V_{HHH} s_H^2 + V_{HHW} s_{HW} + \frac{1}{2} V_{HWW} s_W^2,$$

and substituting from (A.3), it follows that

$$dV_H = \left\{ \left[r + d - y(m) + \frac{we(H)y'(m)}{c} (h_f l + h_e(1-l)) \right] V_H - V_{HH} H s_H^2 - V_{HW} W s_{HW} \right\} dt + \\ + V_{HH} H dh_t + V_{HW} W dw_t \quad (A.4)$$

Suppose $h_f = h_e \stackrel{def}{=} h$ - constant elasticity of $f(\cdot)$, $e(\cdot)$ with respect to H . Then expression (A.4) could be further simplified to the following form,

$$dV_H = \left\{ \left[r + d - y(m) \left(1 - \frac{we(H)y'(m)h}{cy(m)} \right) \right] V_H - V_{HH} H s_H^2 - V_{HW} W s_{HW} \right\} dt + \\ + V_{HH} H dh_t + V_{HW} W dw_t \quad (A.5)$$

which is an equation (3.7) in the text. The procedure for the other state variable, wealth, is completely analogous and yields equation (3.8) in the text.

Appendix B. Solution of the Particular Case with Logarithmic Utility Function

Substituting (3.9) - (3.11) into the Bellman equation (3.6), the following expression is true identically:

$$a \ln \left(\frac{aW}{a_0} \right) + b \ln \left(\frac{bW}{a_0 w a_e H^h} \right) + b h \ln H + b \ln a_f - r \{ a_0 \ln W + b_0 \ln H + g_0 \} + \\ + b_0 \left\{ y \ln \left(\frac{b_0 y}{a_0 c} W \right) - d \right\} + \\ + \frac{a_0}{W} \left\{ rW + w a_e H^h \left(1 - \frac{b}{a_0 w a_e H^h} W \right) - \frac{a}{a_0} W - \frac{b_0 y}{a_0} W \right\} - \\ - \frac{1}{2} b_0 s_H^2 - \frac{1}{2} a_0 s_W^2 = 0$$

Approximating term $\frac{a_0}{W} w a_e H^h$ by the first-order expansion $a_0 w a_e (1 + h \ln H - \ln W)$, and collecting coefficients on constant, $\ln W$, and $\ln H$, we need to find a_0, b_0, g_0 that solve the

following system:

$$a + b - ra_0 + b_0y - a_0wa_e = 0; \quad (B.1)$$

$$-rb_0 + a_0wa_e h = 0; \quad (B.2)$$

$$\begin{aligned} a \ln \frac{a}{a_0} + b \ln \frac{b}{a_0wa_e} + b \ln a_f - rg_0 + b_0y \ln \frac{b_0y}{ca_0} - \\ - b_0d + a_0r - a - b - b_0y + \\ + a_0wa_e - \frac{1}{2}b_0s_H^2 - \frac{1}{2}a_0s_W^2 = 0 \end{aligned} \quad (B.3)$$

The first two equations can be solved for a_0, b_0 using Cramer's formulae:

$$a_0 = \frac{1}{\Delta} \begin{vmatrix} a+b & -y \\ 0 & -r \end{vmatrix}, \quad b_0 = \begin{vmatrix} r+a_e w & a+b \\ wa_e h & 0 \end{vmatrix}, \quad \text{where } \Delta = \begin{vmatrix} r+a_e w & -y \\ wa_e h & -r \end{vmatrix},$$

or, expanding the determinants,

$$a_0 = \frac{a+b}{r+a_e w(1-\frac{yh}{r})}, \quad b_0 = \frac{a_e w(a+b)}{r+a_e w(1-\frac{yh}{r})} \frac{h}{r}.$$

The remaining unknown coefficient g_0 is chosen so that the condition (B.3) is satisfied.

Hence, for the problem (3.1)-(3.3) where the utility function is of the form (3.9), the value function is given by the expression (3.12) in the text.

Appendix C. Derivation of the stochastic accumulation equation for demand for medical services

From the FONC (3.5.c), $m = j \left(\frac{cV_W}{HV_H} \right)$, where $j = (y')^{(-1)}$ is the inverse of the derivative of the

investment function ψ . The previous section established the laws of evolution for V_H, V_W and H ,

so we could apply Itô's lemma again to derive the evolution of m_t . Calculating the derivatives involved in the Itô's lemma expansion and using the fact that $y' = j^{(-1)}$, it is easy to verify that

$$\begin{aligned}
\frac{\partial m}{\partial V_W} &= \frac{j' j^{(-1)}}{V_W}, & \frac{\partial m}{\partial V_H} &= -\frac{j' j^{(-1)}}{V_H}, & \frac{\partial m}{\partial H} &= -\frac{j' j^{(-1)}}{H}, \\
\frac{\partial^2 m}{\partial V_W^2} &= \frac{j' [j^{(-1)}]^2}{V_W^2}, & \frac{\partial^2 m}{\partial V_W \partial V_H} &= -\frac{c}{V_H^2 H} [j' j^{(-1)} + j'], \\
\frac{\partial^2 m}{\partial V_W \partial H} &= -\frac{c}{V_H H^2} [j' j^{(-1)} + j'], & \frac{\partial^2 m}{\partial V_H^2} &= \frac{j^{(-1)}}{V_H^2} [j' j^{(-1)} + 2j'], \\
\frac{\partial^2 m}{\partial V_H \partial H} &= -\frac{j^{(-1)}}{V_H H} [j' j^{(-1)} + j'], & \frac{\partial^2 m}{\partial H^2} &= \frac{j^{(-1)}}{H^2} [j' j^{(-1)} + 2j'],
\end{aligned} \tag{C.1}$$

where j' and $j^{(-1)}$ are evaluated at $\frac{cV_W}{HV_H}$.

Recall that under the assumption $\mathbf{h}_f = \mathbf{h}_e \stackrel{def}{=} \mathbf{h}$ - the constant elasticity of $f(\cdot)$, $e(\cdot)$ with respect to H , the evolution of V_H is given by the expression (3.7);, which is reproduced here for convenience:

$$\begin{aligned}
dV_H &= \left\{ \left[r + d - y(m) \left(1 - \frac{we(H)y'(m)\mathbf{h}}{cy(m)} \right) \right] V_H - V_{HH} H s_H^2 - V_{HW} W s_{HW} \right\} dt + \\
&\quad + V_{HH} H dh_t + V_{HW} W dw_t
\end{aligned} \tag{C.2}$$

and the evolution of wealth by the equation (3.8), which is:

$$dV_W = \left\{ (r - r)V_W - V_{HW} H s_{HW} - V_{WW} W s_W^2 \right\} dt + V_{WH} H dh_t + V_{WW} W dw_t. \tag{C.3}$$

It is easy to verify that

$$\begin{aligned}
\text{var}(dV_H) &= (V_{HH}^2 H^2 s_H^2 + V_{HW}^2 W^2 s_W^2 + 2V_{HH}V_{HW}HWs_H)dt, \\
\text{var}(dV_W) &= (V_{HW}^2 H^2 s_H^2 + V_{WW}^2 W^2 s_W^2 + 2V_{HW}V_{WW}HWs_H)dt, \\
\text{var}(dH) &= H^2 s_H^2 dt,
\end{aligned} \tag{C.4}$$

$$\begin{aligned}
\text{cov}(dV_H, dV_W) &= (V_{HW}V_{HH}H^2 s_H^2 + V_{WW}V_{HH}HWs_{HW} + V_{HW}^2 WHs_{HW} + V_{HW}V_{WW}W^2 s_H^2)dt, \\
\text{cov}(dV_W, dH) &= (V_{HW}H^2 s_H^2 + V_{WW}HWs_{HW})dt, \\
\text{cov}(dV_H, dH) &= (V_{HH}H^2 s_H^2 + V_{HW}HWs_{HW})dt.
\end{aligned}$$

Using Itô's formula,

$$\begin{aligned}
dm &= \frac{\partial m}{\partial t} + \frac{\partial m}{\partial V_H} dV_H + \frac{\partial m}{\partial V_W} dV_W + \frac{\partial m}{\partial H} dH + \\
&+ \frac{1}{2} \frac{\partial^2 m}{\partial V_H^2} \text{var}(dV_H) + \frac{1}{2} \frac{\partial^2 m}{\partial V_W^2} \text{var}(dV_W) + \frac{1}{2} \frac{\partial^2 m}{\partial H^2} \text{var}(dH) + \\
&+ \frac{\partial^2 m}{\partial V_H \partial V_W} \text{cov}(dV_H, dV_W) + \frac{\partial^2 m}{\partial V_H \partial H} \text{cov}(dV_H, dH) + \frac{\partial^2 m}{\partial V_W \partial H} \text{cov}(dV_W, dH)
\end{aligned}$$

After substitution from (C.1)-(C.4), the expression for dm could be simplified under the additional assumption that the utility function and corresponding value function exhibit constant relative

risk aversion. Let by definition $\frac{HV_{HH}}{V_H} = R_{HH}$, $\frac{HV_{WH}}{V_W} = R_{WH}$, $\frac{WV_{WW}}{V_W} = R_{WW}$, and

$R_{WH} = R_{HW}$, be some constants equal to minus the coefficients of relative risk aversion³.

Then after some algebraic manipulation, the following dynamic equation is obtained:

$$dm = \left[j' j^{(-1)} \{-r + n_1\} + j' [j^{(-1)}]^2 \left\{ -\frac{w}{c} e(H)h + n_2 \right\} + j' [j^{(-1)}]^2 n_3 \right] dt + dm_t, \tag{C.5}$$

where

$$\begin{aligned}
n_1 &= s_H^2 (1 + R_{HH})^2 - R_{WW} (s_{HW} + s_W^2) + R_{HW} (R_{HW} s_W^2 - R_{WW} s_W^2 - R_{HW} s_{HW} - s_H^2 + \\
&+ 2R_{HH} s_{HW} + s_{HW}) - R_{WW} R_{HH} (s_{HW} + s_H^2),
\end{aligned}$$

³ See Kihlstrom and Mirman (1974), Karni (1979) for the discussion of multivariate risk aversion.

$$n_2 = \frac{1}{2}R_{WH}^2 s_H^2 + \frac{1}{2}R_{WW}^2 s_W^2 + R_{WH}R_{WW}s_{HW}, \quad (C.6)$$

$$n_3 = R_{HW}^2 \left(\frac{1}{2}s_W^2 - s_{HW} \right) - R_{WW}R_{HH}(s_H^2 + s_{HW}) + R_{HW}(s_{HW} - s_H^2) - \\ - R_{WW}(R_{HW}s_W^2 + s_{HW}) + s_H^2 \left(\frac{1}{2}R_{HH}^2 + 1 \right) + R_{HH}(s_H^2 + R_{HW}s_{HW}),$$

$$\text{and } dm_t = j' j^{(-1)} \{ [R_{HW} - R_{HH} - 1] dh_t + [R_{WW} - R_{HW}] dw_t \}$$

From the definition of the function φ , $j^{(-1)} = y'$, and the theorem about the derivative of the

inverse function⁴, it follows that $j' = [(y')^{(-1)}]' = \frac{1}{y''}$, and $j'' = -\frac{y'''}{[y'']^3}$, where j' and j'' are

evaluated at $\frac{cV_W}{HV_H}$, and y', y'' and y''' are evaluated at m . Assume further that the investment

function is as smooth as necessary, and exhibits constant elasticity with respect to m , constant relative risk aversion, and constant relative prudence⁵, that is,

$$-\frac{my'(m)}{y(m)} = h_1, \quad -\frac{my''(m)}{y'(m)} = h_2, \quad -\frac{my'''(m)}{y''(m)} = h_3, \quad \text{where } h_1 < 0, h_i \in R^+, i = \overline{2,3}. \quad (C.7)$$

Under the assumption above on the investment function, $j' j^{(-1)} = -\frac{m}{h_2}$, $j'' [j^{(-1)}]^2 = \frac{h_1}{h_2} y(m)$,

$j''' [j^{(-1)}]^2 = \frac{h_3}{h_2^2} m$, and equation (C.5) could be simplified to the expression (4.1) in the text.

⁴ Bronshtein and Semendyayev (1964), page 372

⁵ The notion of prudence and properties of the marginal utility functions yielding constant absolute or relative prudence is introduced and studied in Kimball (1990), Blanchard and Mankiw (1988).

⁶ An example of the function with all desired properties is $y(m) = \frac{m^{1-q}}{1-q}$.

Appendix D. Derivation of evolution of marginal value of health and wealth (with insurance)

After substitution of the optimal values from the FONCs (4.8) into the Bellman equation, the following is true identically:

$$\begin{aligned}
 0 = & \max_{c, I, m} \{U(c, f(H)I) - rV + V_t + V_H(y(m) - d)H + V_W(rW + we(H)(1 - I) - c - kcm - \\
 & - (1 - k)c \left[\frac{m}{h_2} \left\{ r - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right] \} + \\
 & + \frac{1}{2} V_{HH} H^2 s_H^2 + V_{HW} HW s_{HW} + \frac{1}{2} V_{WW} W^2 s_W^2 \}
 \end{aligned} \tag{D.1}$$

It is not difficult to establish by partially differentiating the Bellman equation (D.1) with respect to H , and taking into account FONCs (4.8.a-c), that

$$\begin{aligned}
 (y(m) - r - d)V_H + U_I f'(H)I + V_{Ht} + V_{HH}(y(m) - d)H + V_{WH}(rW + (1 - I)we(H) - c - kcm - \\
 - (1 - k)c \left[\frac{m}{h_2} \left\{ r - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right] \} + \\
 + V_W \{ we'(H)(1 - I) + (1 - k)c \frac{h_1}{h_2} y(m) \frac{w}{c} e'(H)h \} + V_{HH} H s_H^2 + V_{HW} W s_{HW} + \\
 + \frac{1}{2} V_{HHH} H^2 s_H^2 + V_{HHW} HW s_{HW} + \frac{1}{2} V_{HWW} W^2 s_W^2 = 0
 \end{aligned} \tag{D.2}$$

Equation (D.2) could be further simplified by noting that $U_I f'(H)I = \frac{V_W we(H) f'(H)}{f(H)} I$, and

collecting terms containing V_W yields the following expression:

$$\begin{aligned}
 (y(m) - r - d)V_H + V_{Ht} + V_{HH}(y(m) - d)H + V_{HW}(rW + (1 - I)we(H) - c - kcm - \\
 - (1 - k)c \left[\frac{m}{h_2} \left\{ r - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right] \} \\
 + V_W we(H) \left(\frac{f'(H)}{f(H)} I + \frac{e'(H)}{e(H)} (1 - I) + (1 - k) \frac{h_1}{h_2} y(m) h \frac{e'(H)}{e(H)} \right) + \\
 + \frac{1}{2} V_{HHH} H^2 s_H^2 + V_{HHW} HW s_{HW} + \frac{1}{2} V_{HWW} W^2 s_W^2 + V_{HH} H s_H^2 + V_{HW} W s_{HW} = 0
 \end{aligned} \tag{D.3}$$

Denoting $h_f = -H \frac{f'(H)}{f(H)}$ and $h_e = -H \frac{e'(H)}{e(H)}$, with $h_f = h_e = h$ by assumption, equation (D.3) is

reduced to the following:

$$\begin{aligned}
& [y(m) - r - d]V_H + V_{Ht} + V_{HH}(y(m) - d)H + V_{WW}(rW + (1 - I)we(H) - c - kcm - \\
& - (1 - k)c \left[\frac{m}{h_2} \left\{ r - n_1 + \frac{h_3}{h_2} n_3 \right\} - \frac{h_1}{h_2} y(m) \left\{ \frac{w}{c} e(H)h - n_2 \right\} \right] + \\
& - \frac{V_W we(H)h}{H} \left\{ 1 + \frac{h_1}{h_2} (1 - k)y(m)h \right\} + \frac{1}{2} V_{HHH} H^2 s_H^2 + \\
& + V_{HHW} H W s_{HW} + \frac{1}{2} V_{HWW} W^2 s_W^2 + V_{HH} H s_H^2 + V_{HW} W s_{HW} = 0
\end{aligned} \tag{D.4}$$

Using Itô's lemma,

$$dV_H = V_{Ht} dt + V_{HH} dH + V_{HW} dW + \frac{1}{2} V_{HHH} s_H^2 + V_{HHW} s_{HW} + \frac{1}{2} V_{HWW} s_W^2, \tag{D.5}$$

and substituting from (D.4), it follows from the equation (D.5) that

$$\begin{aligned}
dV_H = & \{ [r + d - y(m)]V_H + \frac{V_W we(H)h}{H} \left\{ 1 + \frac{h_1}{h_2} (1 - k)y(m)h \right\} - \\
& - V_{HH} H s_H^2 - V_{HW} W s_{HW} \} dt + V_{HH} H dh_t + V_{HW} W dw_t
\end{aligned} \tag{D.6}$$

which is the equation (4.9) in the text.

Performing the same procedure for another state variable, wealth, it is easy to establish that

$$dV_W = \{ (r - r)V_W - V_{HW} H s_{HW} - V_{WW} W s_W^2 \} dt + V_{WH} H dh_t + V_{WW} W dw_t, \tag{D.7}$$

which corresponds to the equation (4.10).

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