Wage Differential, Trade, Productivity Growth and Education

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Abstract

There is a large literature on the link between wage differential, international trade and productivity growth. The theoretical and empirical research is mainly based on the Heckscher-Ohlin-Samuelson framework and on the cases of a large country. More comprehensive theoretical models are needed to guide further empirical research. This paper contributes to the debate by providing a dynamic intertemporal general equilibrium (DIGE) model incorporating endogenous skill formation. The result tends to support the argument that trade has a responsibility for wage differential. A cut in government education investment tends to raise wage differential. Productivity growth at best causes wage differential in the short run. From a theoretical perspective it is unclear whether productivity growth raises wage differential in the long run once the accumulation of skills is endogenized.

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1. Introduction

The relationship between international trade and wages has been the focus of an expanding literature. One viewpoint proposes that international trade is responsible for the wage differential between skilled and unskilled labor. For example Wood (1995) suggests that trade is the main cause of wage inequality[1]. The other view is that trade is not the main cause of this phenomenon. For example Krugman and Lawrence (1994) point out that skill-biased productivity growth is the main cause of increasing wage inequality[2]. This debate is ongoing[3] in the OECD economies, and extensive research has contributed to large country cases. Only limited research has been based on small open economies[4]. Whether for a big or small economy study, the theoretical and empirical research is largely based on the Heckscher-Ohlin-Samuelson framework and the econometric methodology. More comprehensive theoretical models are needed to guide further empirical research.

This paper constructs a dynamic intertemporal general equilibrium (DIGE) model of a small open economy with three-sectors (exports, imports, and non-traded), two kinds of labor (skilled and unskilled), and three-agents (firms, households, and government). To model the process of skill formation is essential when considering both skilled and unskilled labor. Education is the fundamental input to skill formation[5]. Hence, education production and consumption play important roles in this model. A general picture of the modeling methodology is presented in Section 2. Section 3 illustrates the theoretical framework. Section 4 frames the simulation results and Section 5 concludes.

2. Modeling Methodology

Firms employ physical capital, skilled and unskilled labor, to produce three types of goods, i.e. exports, imports, and non-traded[6], and sell these goods to households for consumption, to government for education capital investment, and to themselves for physical capital investment. The export good is sold abroad based on foreign demand, and some portion of the import goods comes from abroad. Households supply unskilled labor to firms and skilled labor to both firms and government in order to gain wages. Households also own the physical capital and earn financial dividends to finance goods consumption from firms and purchases of education from government. Households also consume leisure with an opportunity cost of not working, and maximize utility by distributing consumption optimally on both goods and leisure. The government buys the goods from firms and transforms it into education capital, together with hiring skilled labor to produce education. The government also balances its budget by collecting labor

[1] Murphy and Welch (1991), Leamer (1992), and Sachs and Shatz (1994) support this strand.

[2] Katz and Murphy (1992), Berman, Bound, and Griliches (1994), and Slaughter and Swagel (1997) support this point.

[3] Krugman (1995) debating Leamer (1994) is an impressive example.

[4] Chen and Hsu (1998) is a recent contribution for a case study of Taiwan.

[5] Bhagwati and Srinivasan (1977) indicate education confers skills.

[6] Three types of goods are also characterized as skilled labor intensive, unskilled labor intensive, and capital intensive.

income tax and selling education to households to finance its spending on education capital investment and skilled labor hiring. Physical capital, education capital and financial assets accumulate through time.

The numbers of workers in the two categories of skilled and unskilled labor varies along time. In addition to government investment in education and training facilities, the motivation of households to pursue higher skills is an important factor when considering changes in the proportions of labor in the different categories. The main incentive for acquiring skills is higher future income. This motivates candidates to forsake current unskilled wage, and to invest, by time consumption and direct cost (tuition), in upgrading their education level[7]. To model the transformation of unskilled into skilled labor, the method assumes a representative agent. A portion of the person serves as skilled labor and the remaining portion as unskilled labor. In each period of time, this representative person makes a choice to invest in education to gain skill, which is subject to an increasing convex cost of adjustment function. The skilled labor faces no transformation cost to work as unskilled labor. The equilibrium skill formation depends on the fixed skill formation, skill depreciation rate, and the skill stock as well as the demand for skilled labor. Due to the assumption of the mobility of labor internally, there is a single nominal wage for skilled workers and a single nominal wage for unskilled workers.

The elasticity of the skilled labor supply, with respect to the wage of skilled labor, is greater than zero, since the supply of skilled labor is not fixed. It is less than infinity in the short run, because the transformation from unskilled to skilled labor is not fully free. Some skills are specific or patented, and training facilities are not always established quickly. It is also not perfectly elastic in the long run. There are four main reasons proposed on this point by Wood (1994), but only two are appropriate in this model[8]. First, the imperfection of capital markets, which means the opportunity to acquire loans for schooling, is not equal for all unskilled labor, raising the issue of unequal availability of skill acquisition. While, becoming a full time student means only a decrease in savings for some, it can mean starvation for others. Second, variations in trainability amongst unskilled labor arise from differences in innate ability and family background (Phelps Brown, 1977). These two factors provide the accuracy of embodying a positive and limited elasticity of skilled labor supply in the model.

3. The Model

This model follows the general approach of the G-Cubed model (McKibbin and Wilcoxen (1999)) incorporating the first attempt to embed education and endogenous skill formation issue.

3.1 Firms

[7] In the ensuing model, education is the necessary channel for acquiring skills. Learning by doing is treated as a skill improvement in the same category, not jumping into a high level. Spill over effect is embedded into the education investment function.

[8] The other two reasons are that skill transformation is increasing returns, and external, which are proper to explain the enduring gap of skill levels between North and South.

There are three representative firms in the economy: an export sector, an import sector, and a non-traded sector. A Cobb-Douglas production function is employed[9],

(3.1) $$Q_i = A_{Qi} \cdot K_i^{\alpha i} \cdot {L_{s,i}^F}^{\beta i} \cdot L_{u,i}^{1-\alpha i-\beta i},$$

where Q_i is the gross output, A_{Qi} is the sectoral technology parameter, i is the sector index and i = 1, 2, 3. The three inputs are capital K_i, skilled labor $L_{s,i}^F$, and unskilled labor $L_{u,i}$, and α_i, β_i, $(1- \alpha_i - \beta_i)$ are, respectively, the shares of employment of capital, skilled labor, and unskilled labor in production.

The sector with a larger share of skilled labor is defined as a relatively skilled intensive sector. The sector with a larger share of unskilled labor is defined as a relatively unskilled intensive sector. And the sector with a larger share of capital is defined as relatively capital intensive. To initialize the model, it is assumed sector 1 is relatively skilled intensive, sector 2 is relatively unskilled intensive, and sector 3 is relatively capital intensive. It is also assumed this small open economy exports good 1, imports good 2 and good 3 is non-traded. That is, the representative firms export the relatively skilled intensive and import the relatively unskilled intensive goods. Table 1 illustrates this characteristic.

Table 1 Sector Characteristic

Sector	Factor Intensive	Trade
1	Skilled	Export
2	Unskilled	Import
3	Capital	Non-traded

Export (X) is a function of foreign income (Y^*) and the inverse of terms of trade as follows,

(3.2) $$X = \left(\frac{P_2}{P_1}\right)^{\rho} \cdot Y^*,$$

where ρ is a parameter. Equation (3.2) states that exports of the domestic country are boosted by an increase in foreign income, a decline in export price (P_1), and a rising import price (P_2). Exports decrease when the opposite is true. This model makes a case that the export good is an imperfect substitute for the foreign good, whereas the import good is a perfect substitute for the domestic good. To simplify, it is assumed that trade is balanced in all periods.

The capital accumulation in each sector depends on the rate of fixed capital formation J_i and the rate of depreciation δ_i,

(3.3) $$\frac{dK_i}{dt} = J_{t,i} - \delta_i \cdot K_{t,i}.$$

[9] Every variable implicitly carries a subscript of time.

Under the assumption of rising marginal costs of installation in the investment process, the total investment expenditure I_i in sector i is

(3.4) $$I_i = J_i \cdot [1 + (\frac{\phi_i}{2} \cdot \frac{J_i}{K_i})],$$

where ϕ_i is a positive parameter, and $(\phi_i/2)(J_i/K_i)$ is the unit costs of adjustment, which is assumed to be a linear function of the rate of capital formation.

An optimal firm maximizes intertemporal profit, and the inputs of L_s^F, L_u and J are chosen subject to equations (3.3) and (3.4) by solving the current value Hamiltonian function with λ_i is the shadow price of capital. The first order conditions state that the real return to factors is equal to its marginal productivity under perfect competition. The shadow price of capital is greater than one due to the adjustment cost. By solving the first-order differential equation and applying the transversality condition, the shadow price of capital becomes

(3.5) $$\lambda_i(t) = \int_t^\infty (Q_{Ki} + \Theta_i)\, e^{-(r+\delta_i)\cdot s} \cdot ds,$$

where $\Theta_i = (\phi_i/2)(J_i/K_i)^2$. Equation (3.5) states that the shadow price of capital is equal to the present discounted value of future marginal products. It consists of two parts, Q_{Ki} is the marginal product of capital and Θ_i is the marginal product of capital in reducing adjustment costs in investment at each point in time. Therefore, λ_i is the increment to the real value of the firm from one unit of installed capital at time t.

The labor demand function for both skilled and unskilled labor[10] follows

(3.6) $$\ln L_{j,i} = \ln Z_i + \ln Q_i - \ln W_j + \ln P_i,$$

where Z is the constant term and j = s,u. From equation (3.6), it is obvious, in a partial equilibrium, labor demand has a positive relationship with production and a negative relationship with wage. An increase in the price of good i raises the demand for both skilled and unskilled labor in sector i, because firms are willing to produce more of good i than previously.

3.2 Household

The household distributes after-tax labor income and dividends to consumption of the three goods, financial asset accumulation for future consumption and education investment, together with the choice of leisure, to maximize utility. The objective function of the household, the intertemporal budget constraint, a dynamic accumulation of skilled labor, the financial assets ownership, and education investment are shown as follows,

[10] For capital, a similar functional form is used. However, it is not of concern here.

(3.7) Max. $\int_0^\infty U(C_{1,t}, C_{2,t}, C_{3,t}, l_t) \cdot e^{-\theta t} \cdot dt$

Subject to

(3.8) $$\frac{dF_t}{dt} = r_t \cdot F_t + (1-\tau_t) \cdot (\frac{W_s}{P_2} \cdot L_{s,t} + \frac{W_u}{P_2} \cdot L_{u,t}) - [(\frac{P_1}{P_2}) \cdot C_{1,t} + C_{2,t} + (\frac{P_3}{P_2}) \cdot C_{3,t} + (\frac{P_{E,t}}{P_2}) \cdot S_{E,t}],$$

(3.9) $$\frac{dL_{s,t}}{dt} = J_{s,t} - \delta_s \cdot L_{s,t},$$

(3.10) $$F_t = (\frac{P_1}{P_2}) \cdot \lambda_{1,t} \cdot K_{1,t} + \lambda_{2,t} \cdot K_{2,t} + (\frac{P_3}{P_2}) \cdot \lambda_{3,t} \cdot K_{3,t},$$

(3.11) $$S_{E,t} = J_{s,t} \cdot [1 + \frac{\Phi}{2} (\frac{J_{s,t}}{L_{s,t}})],$$

where $C_{i,t}$ is the consumption of goods, l_t is the leisure taking, θ is the rate of time preference, F_t is the financial assets, $P_{E,t}$ is the price of one unit of education, $S_{E,t}$ is the amount of education buying, $J_{s,t}$ is the fixed skill formation, δ_s is the depreciation rate of skill, and Φ is the adjustment cost parameter.

Equation (3.8), the budget constraint, states that the accumulation of the household's financial assets depends on financial dividends, total after-tax labor income and total spending on goods and education. Equation (3.9) states that the net skill formation is the skill depreciation subtracted from the fixed skill formation. Equation (3.10) shows the contents of financial assets, which includes the value of capital in each sector, so-called equity. Equation (3.11) states that education investment depends on fixed skill formation and an adjustment cost function. The adjustment cost reflects the foregone production and relies on the ratio of fixed skill formation to skilled labor. If skilled labor is increasing, the adjustment cost is decreasing. It is plausible due to the spill over effect among labor. Leisure is defined as the remaining time after deducting total labor hours.

The current value Hamiltonian function is employed to solve the above autonomous two-state variables system with μ_1 and μ_2 the shadow prices respective to the financial assets and skill. The first order conditions imply $\mu_1 = MU_1/P_1 = MU_2/P_2 = MU_3/P_3$, where MU_i stands for the marginal utility of consuming good 1, good 2 and good 3. It demonstrates that marginal utility should be the same on consuming each good to achieve optimality. The shadow price of skill is greater than the shadow price of the financial asset, because the total cost of forming a unit of skill is greater than that of accumulating one unit of financial asset, due to the adjustment cost of skill formation. If the shadow price of skill is not greater than that of the financial asset, the household would like to defer spending on skill formation, instead of accumulating it into financial assets for future consumption.

Applying the transversality condition to the shadow price of skill, μ_2, results in

(3.12) $$\mu_2(t) = \int_t^{\infty} \{\mu_1 \cdot [(1 - \tau_1) \cdot \frac{W_s}{P_2} + \frac{P_E}{P_2} \cdot \frac{\Phi}{2} \cdot (\frac{J_s}{L_s})^2] + U_{Ls,\,t} \} \cdot e^{-(\theta + \delta s) \cdot t} \cdot dt,$$

where $U_{Ls,\,t}$ is the partial derivative of utility function to skilled labor. The transversality condition asserts that, in an infinite time horizon, the present value of the shadow price of one additional skilled labor formation is equal to zero. It eliminates the case of infinite accumulation of skilled labor. This assertion is plausible because the variation of the skilled wage initialized by the movement of the skilled labor supply and demand can tie down infinite accumulation of skilled labor. Equation (3.12) states that the shadow price of skill is equal to the present discounted value of future marginal utility. The first component of the shadow price of skill contains the marginal utility of consuming goods, the after-tax skilled wage, and the reduction of adjustment cost in education investment. It provides the gross increment of utility the household can get from supplying one additional unit of skilled labor. The second part is the marginal disutility of offering one unit of skilled labor. Combining these two ends with the net utility the household can achieve by supplying one unit of skilled labor substantiates the essence of μ_2.

To derive consumption on goods and leisure, a Cobb-Douglas utility function is assumed, resulting in

(3.13) $$C_i = \frac{\gamma_i \cdot U \cdot P_2}{\mu_1 \cdot P_i},$$

(3.14) $$l = \frac{(1-\gamma_1-\gamma_2-\gamma_3) \cdot U \cdot P_2}{(1-\tau) \cdot \mu_1 \cdot W_u}.$$

Equations (3.13) and (3.14) illustrate that consumption on goods and leisure are determined by the ratio of utility share and the product of shadow price μ_1 and prices, or the net earnings of supplying one unit of unskilled labor. It states, ceteris paribus, when the price or net unskilled earnings is increasing, the household decreases demand for goods or leisure. Also, when consumption goes up with fixed prices, the marginal utility μ_1 decreases, i.e., the law of diminishing return. This is an outcome of the concave utility function. Equation (3.14) also shows that, if income tax increases, leisure increases. Hence, total labor hours decrease. This corroborates that heavy taxation lessens the motivation for working in a partial equilibrium.

3.3 Government

The role of government as an education supplier is essential. This model attempts to capture the reality of government supplying education in consideration of externalities. The government collects income tax (Tax_t), and sells education ($P_{E,t} \cdot S_{E,t}$) to finance total spending, and also has to buy good 1 ($I_{E,1}^G$), good 2 ($I_{E,2}^G$), and good 3 ($I_{E,3}^G$) to produce education capital ($K_{E,t}$). Total government investment on education capital is represented by $I_{E,t}^G$. It is assumed that $I_{E,t}^G$ is exogenously controlled by the government.

To produce education, the government employs skilled labor (L_s^G) as well as using education capital,

(3.15) $$S_{E,t} = K_{E,t}^{\xi} \cdot L_s^{G^{1-\xi}},$$

where ξ is the input share parameter.

The accumulation of education capital is followed by the total investment of government subtracting the depreciation,

(3.16) $$\frac{dK_{E,t}}{dt} = I_{E,t}^G - \delta_E \cdot K_{E,t},$$

where δ_E is the rate of depreciation and $I_{E,t}^G$ is defined as follow,

(3.17) $$I_{E,t}^G = \frac{1}{P_E^G} \cdot (P_1 \cdot I_{E,1}^G + P_2 \cdot I_{E,2}^G + P_3 \cdot I_{E,3}^G),$$

where P_E^G is the weighted price index defined as below,

(3.18) $$P_E^G = P_1^{\varepsilon_1} \cdot P_2^{\varepsilon_2} \cdot P_3^{\varepsilon_3},$$

where ε_i is the weight of this pooled index.

Government spending on each good for education investment follows $I_{E,i}^G = \varepsilon_i \cdot \frac{P_E^G \cdot I_E^G}{P_i}$.

To assure the model is consistent, as well as the economy is in equilibrium, the rule of demand equal to supply is applied on both sides of the goods and factors market.

3.4 Steady State

Table 2 summarizes this model in steady state. It should be highlighted that the rate of time preference is equal to the rate of interest in the steady state. If the rate of time preference is greater than the interest rate, the household will decumulate financial assets instead of increasing consumption. This will raise the interest rate until it reaches the level of time preference. The adjustment reverses if the interest rate is greater than the rate of time preference. Hence, those two rates have to be equal in equilibrium[11]. The shadow price of capital is well known as marginal *Tobin's* q, noted as q_i^M. In steady state, it follows

(3.19) $$q_i^M = 1 + \delta_i \cdot \phi_i .$$

[11] This result is consistent with the argument in chapter two of Blanchard and Fischer (1989).

Table 2 Model in Steady State

Equations
$Q_i = A_{Qi} \cdot K_i^{\alpha i} \cdot L_{s,i}^{F\ \beta i} \cdot L_{u,i}^{1-\alpha i-\beta i}$
$J_{t,i} = \delta_i \cdot K_{t,i}$
$I_i = J_i \cdot (1 + \phi_i \delta_i/2)$
$Q_{i,Ls} = W_s/P_i$
$Q_{i,Lu} = W_u/P_i$
$\lambda_i = 1+ \phi_i \cdot \delta_i$
$Q_{Ki} = (r + \delta_i)\lambda_i - \phi_i \cdot \delta_i^2/2$
$P_{2,t} \cdot M_t = P_{1,t} \cdot X_t$
$X = (P_2/P_1)^{\rho} \cdot Y^*$
$0= r_t \cdot F_t + (1-\tau_t) \cdot [(W_s/P_2) \cdot L_{s,t}+ (W_u/P_2) \cdot L_{u,t}] - [(P_1/P_2) \cdot C_{1,t} + C_{2,t} + (P_3/P_2) \cdot C_{3,t} + (P_{E,t}/P_2) \cdot S_{E,t}]$
$J_{s,t} = \delta_s \cdot L_{s,t}$
$F_t = (P_1/P_2) \cdot \lambda_{1,t} \cdot K_{1,t} + \lambda_{2,t} \cdot K_{2,t} + (P_3/P_2) \cdot \lambda_{3,t} \cdot K_{3,t}$
$I_{E,t} = J_{s,t} (1+\Phi \cdot \delta_s/2)$
$l_t = T - L_{s,t} - L_{u,t}$
$U_{Ci} = (P_i/P_2) \cdot \mu_1$
$U_{Lu,t} = -\mu_1 \cdot (1- \tau) \cdot W_u/P_2$
$\mu_2 = \mu_1 \cdot P_E \cdot (1+\Phi \cdot \delta_s)/P_2$
$r_t = \theta$
$U_{Ls} = (\theta + \delta_s) \cdot \mu_2 - \mu_1 \cdot [(1 - \tau) \cdot W_s + P_E \cdot (\Phi \cdot \delta_s^2)/2]/P_2$
$S_E = K_E^{\xi} \cdot L_s^{G^{1-\xi}}$
$I_E^G = \delta_E \cdot K_E$
$I_{E,t}^G = (P_1 \cdot I_{E,1}^G + P_2 \cdot I_{E,2}^G + P_3 \cdot I_{E,3}^G)/P_E^G$
$P_E^G = P_1^{\varepsilon_1} \cdot P_2^{\varepsilon_2} \cdot P_3^{\varepsilon_3}$
$P_E^G \cdot I_E^G + W_s \cdot L_s^G = \tau \cdot (W_s \cdot L_s + W_u \cdot L_u) + P_E \cdot S_E$
$Q_{1,t} - X_t = C_1 + I_{E,1}^G + I_1$
$Q_{2,t} + M_t = C_2 + I_{E,2}^G + I_2$
$Q_{3,t} = C_3 + I_{E,3}^G + I_3$

Hence, without subsidy of investment, the marginal *Tobin's* q is greater than 1 when an adjustment cost exists, and equal to 1 when the adjustment cost is zero[12]. Similarly, the

[12] After the subsidy of investment is embedded, the marginal *Tobin's* q is smaller when the rate of subsidy is higher. That is to say, if the government subsidizes investment with a higher rate, the investment cost is lower and will encourage firms to invest.

marginal $Tobin's$ q of skill[13], noted as q_s^M, has a relationship with the shadow price μ_2. That is $q_s^M = \mu_2$. Therefore, in steady state,

$$q_s^M = \mu_1 \cdot P_E \cdot (1 + \delta_s \cdot \Phi). \quad (3.20)$$

Equation (3.20) states that, the marginal $Tobin's$ q of skill is equal to the shadow price of financial assets, i.e., the marginal utility of consuming goods, times the total cost of one unit of skill formation. It asserts that skill generates higher wages, hence, more utility. The product of education price and the bracket is equivalent to the real price of upgrading one unit of skill. Hence, the multiple term on the right-hand side is the marginal utility the household can have after gaining one unit of skilled labor. To maximize utility, the household will invest on skill formation until its generation of marginal utility is equal to its marginal cost.

3.5 Wage Differential

This model provides a neat form of the wage differential in steady state,

$$W_s = W_u + \frac{P_E}{(1-\tau)} \cdot (\theta + \theta \cdot \Phi \cdot \delta_s + \delta_s + \frac{1}{2} \cdot \Phi \cdot \delta_s^2). \quad (3.21)$$

The expression of equation (3.21) is independent of the functional form of utility and production function[14]. It provides a rigorous theoretical result of wage differential. The relationship between the skilled and unskilled wage depends on the rate of time preference, the depreciation rate of skill, the skill adjustment cost parameter, the tax rate, and the price of education. A higher skill adjustment cost, skill depreciation rate, or time preference, all tend to boost wage differential. The reason for a higher skill adjustment cost and skill depreciation rate raising wage differential is straightforward, whereas the case for a larger time preference is complicated. The rate of time preference counts because an investment in skill formation takes time to repay. A larger time preference involves a larger adjustment cost for skill formation, therefore a patient household will expect a higher skilled wage. Related to the wage differential equation, a transition mechanism is as follows. A larger time preference means a higher interest rate in the long run, and therefore, a larger financial income. This motivates the household to enjoy more leisure and work less, both as skilled and unskilled labor. The lower demand for education decreases the education price. Due to higher investment costs, a contraction in all three sectors pushes up goods prices and pushes down both the skilled and unskilled wage. The government raises the tax rate to finance its expenditure on goods. Overall, a larger time preference leads to wage differential by pushing down the unskilled wage more than the skilled wage. Hence, even though the firm plays an implicit role in Equation (3.21), its effect could be deterministic. A productivity growth from the firm side pushes down goods prices. This motivates the government to cut the tax rate and decreases wage differential. The government plays an important role in wage differential

[13] It can be regarded as human capital.
[14] A detailed proof is available from the author.

in the context of the education price and taxation. The government controls education investment, and therefore, impinges on education price. If the government increases education investment, thereby decreasing the education price, it can cause convergence of wage differential. A cut in government education investment leads to enlarging wage differential. The factor of trade is not explicitly shown in Equation (3.21) and its effect is transmitted from production to wages through the education price. The intuition is, if exports are skilled intensive, an increase in exports initiates a larger demand for skilled labor. This boosts the education price and enlarges wage differential. What matters in a general equilibrium is the interactive effect of the education price and tax rate. Intuitively, a higher tax rate makes it possible for government to lower the education price. In steady state, since skill formation catches up with the skill demand, a decreasing demand of education drives down the education price. A more accurate policy implication should be illustrated by the simulation in the next section.

The wage differential equation illustrates both the importance and the transmission channel of education in wage differential. This substantiates the inclusion of education in the debate on wage differential, in addition to the traditional argument of trade and productivity growth. The government, as an education supplier and tax collector, has the opportunity to control wage differentials to a certain extent.

4. Simulation Results

A number of simulation are considered in this section in order to explore the long run and short run relationships between variables in the model. The exogenous variables are technology, time, government investment on education, and foreign income. All other variables are endogenous. The setting of the parameters and exogenous variables is presented in the Appendix. The experiment is to investigate the transition of all endogenous variables between steady states in five cases: a 10% improvement in technology in each sector, a 10% increase in government education investment, and a 10% increase in foreign income or exports. It should be noted that this model simultaneously provides the cases of total factor productivity (TFP) and input efficiency growth in each sector. The dynamic path of input efficiency shock is not shown because it is similar to the shocks of TFP[15]. An aggregation of three sectors leads to a case of factor biased productivity growth. This model allows the skill supply and demand to determine the skilled wage, rather than the scenario in Krugman and Laurence (1994), which asserts skill demand dominates the jump of skilled wage[16].

This framework provides a total of thirty cases of different factor intensity in each sector

[15] This model can easily pull in the input efficiency factor. The production function becomes $Q_i = A_{Qi}\cdot(c_{k,i}\cdot K_i)^{\alpha i}\cdot(c_{s,i}\cdot L_{s,i}^F)^{\beta i}\cdot(c_{u,i}\cdot L_{u,i})^{1-\alpha i-\beta i}$, where $c_{j,i}$, j = k, s, u, represents the relative efficiency of factor K, L_s, L_u in sector i. Obviously, the upgrade of input efficiency reaches a similar result, but in a different level of TFP growth.

[16] To adapt this model to the scenario in Krugman and Laurence (1994), either to increase the adjustment cost of skill to a fairly high extent to make it very hard for households to transform skill, or to increase the shares of skilled labor in all sectors to force the skill demand dominating the skilled wage.

combining with trade characteristic and shocks. As the model is very robust[17], only one case is selected to show the detailed economic mechanism, the export sector which is skilled intensive called sector 1, the import sector which is unskilled intensive called sector 2, and the non-traded sector which is capital intensive called sector 3. Table 3 summarizes the steady state results as the percentage change of each endogenous variable responding to each shock. Figures 1 and 2 illustrate the dynamic paths of some key variables[18] with technology shocks and government education and foreign income shocks, respectively. Every variable is expressed as percentage change relative to the baseline, except for the interest rate and tax rate which are expressed as absolute percent. The horizontal and vertical axes stand for the year and percentage, respectively. The analysis contains steady state mechanism, which a computable general equilibrium (CGE) model can usually provide, and dynamic mechanism, which comes from the benefit of a DIGE model.

4.1 Technology Shock in Sector 1

An upgrading of technology in sector 1 leads to a rise in both exports and domestic demand for good 1. In the short run, the household drops consumption of goods 2 and 3, due to their relatively expensive prices. However, in the long run, the household benefits from a technology upgrade and consumes more of all three goods. In the short run, the skilled wage jumps due to increasing demand and the scarcity of skilled labor because skill formation takes time. The higher skilled wage motivates the household to invest more on education, boosting the price of education, and inducing the government to hire more skilled labor to produce more education. In the short run, the unskilled wage drops due to an excess of unskilled labor. In sector 1, both skilled and unskilled labor increase in the short run to support the increasing demand domestically and externally. This is different from the long run transition, because over time, a technology upgrade replaces labor. Thus, in the long run, sector 1 expands and also releases some skilled and unskilled labor. Sector 2 expands, because imports from the foreign country are cut due to the expensive relative price. With sector 2 being a relatively unskilled intensive sector, skill formation and skilled labor have to decline to eliminate oversupply. This induces the skilled wage to decline in the long run. Since more unskilled labor are demanded, the unskilled wage rises along with the amount of unskilled labor[19]. The non-traded sector contracts, due to the lower overall demand, mainly from government cuts. Since real wages, defined here as wages deflated by the pooled price index P_E^G[20], increase due to the drop of price index, the opportunity cost of leisure jumps. Hence, the household drops some leisure. In the long run, the decline in demand for skilled labor and the declined skilled wage contract education investment, and push down the price of education. The

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[17] A sensitivity test on these thirty cases indicates this model is very robust on wage differential issues. A full set of sensitivity test is available from the author.

[18] A full set of dynamic paths is available from the author.

[19] This phenomenon, skilled wage dropping and the unskilled wage rising along with the relative price of good 1 decreasing, corroborates the Stolper-Samuelson theorem in the Hechscher-Ohlin model, if the third sector is ignored.

[20] The pooled price index based on equal weights is a type of wage deflator. Other types of deflators could be used, for example, an index constructed by goods′ prices and education prices, with weights based on goods consumption and education taking shares on total household expenditure. In this simulation, the large drop of the good 1 price has dominated the drop in the price index.

government hires less skilled labor because of the lower education demand, and also cuts the rate of taxation, due to lower spending on goods and hiring skilled labor. In the short run, the interest rate drops due to the glut of capital. It returns to the rate of time preference in the long run.

To summarize the variation of sectors and policy implications, a technology upgrade in an export skilled intensive sector expands both the export skilled intensive and import unskilled intensive sectors, but contracts the non-traded capital intensive sector and education production. Above all, a technology upgrade in the skilled intensive sector enlarges wage differential in the short run and reduces the wage differential in the long run.

4.2 Technology Shock in Sector 2

A technology shock in sector 2 has a basic difference with that in sectors 1 and 3. Since the price of good 2 is set by the world price, due to its perfect substitution to the imports, a technology improvement in sector 2 does not drive down the price of good 2, but instead pulls up other prices.

A technology improvement in sector 2 leads to a cheaper relative price of good 2 and a surge in domestic demand. More imports inflow to compensate for domestic deficiency. Both the export and non-traded sectors contract, resulting from the lower overall demand and the higher production cost, mainly due to a rising terms of trade, government cuts and resources flowing into sector 2. Due to its contraction, sector 1 dominates labor mobility by releasing both types of labor. The increased production of sector 2 does not fulfill the jumping demand, therefore more labor flows into sector 2 to produce the required amount. In the short run, to support the increasing domestic demand, both the skilled and unskilled labor in sector 2 jumps. These increased rates of labor slow down over time, due to a technology improvement that supplants labor in the long run. In the short run, the rise in the skilled wage is due to the cut in the skilled labor supply. The unskilled wage is boosted because the demand in sector 2 is higher than the released quantity of unskilled labor from sectors 1 and 3. In real terms, the unskilled wage is rising more than the skilled wage, motivating the household to serve more as unskilled labor. The household becomes better off receiving the higher wages and consumes more on each good, and accumulates more financial assets. The economy ends up with less skilled labor and an increasing skilled wage. The unskilled wage also keeps rising because of the large amount of unskilled labor demanded by sector 2. Both increasing wages push up the opportunity cost of leisure and stimulate the household to work more. The government cuts tax rate, due to lower spending on hiring skilled labor and to the higher price of selling education. In the short run, the interest rate increases due to the decreasing aggregate capital formed. It returns to the rate of time preference in the long run.

To summarize the change of sectors and policy implications, a technology upgrade in an import unskilled intensive sector expands its own sector, but contracts all other sectors. An important result to be highlighted here is that an expansion from a traded sector contracts the non-traded sector. Above all, a technology upgrade in the unskilled intensive sector reduces wage differential in the short run. In the long run, wages still sustain convergence, but to a small extent relative to the short run response.

4.3 Technology Shock in Sector 3

In the short run, due to a fall in the price of good 3, both the household and the government buy more of good 3 and less of goods 1 and 2. To support the increasing demand, both skilled and unskilled labor in sector 3 increases. Over time, the increased rates of labor either slow down or drop below the baseline. In the short run, exports slightly decrease and imports slightly increase. This variation reverses in the long run, due to a cheaper relative price 1. The reduction in government spending on good 2 dominates a contraction in sector 2. In the short run, the skilled wage jumps due to both the increasing demand and the shortage of skilled labor. The higher skilled wage stimulates the household to invest more on education, boosting the education price. Sectors 1 and 2 employ less of both skilled and unskilled labor, due to the higher wages and lower production. Since sector 3 hires more skilled than unskilled labor[21], a glut of unskilled labor

[21] This is the case of this model in calibration.

results in a fall of the unskilled wage. In the long run, this technology upgrade benefits the household by higher consumption of all three goods, especially good 3. The household also increases education investment and financial assets accumulation. Finally, this economy ends up with an excess of skilled labor pushing down the skilled wage. This leads sector 2 to replace some unskilled with skilled labor and the expanding sector 3 to cut unskilled labor hiring. Since the increasing demand from the household and foreigners for good 1 is greater than the decreasing government demand, sector 1 expands and recruits more input. Unskilled labor hiring in sector 1 dominates the increase of unskilled labor and boosts the unskilled wage. Due to the higher opportunity cost, the household decreases leisure. Since the education price goes up and the skilled wage goes down, the government cuts the rate of taxation to balance its budget.

To summarize the variation of sectors and policy implications, a technology upgrade in the non-traded capital intensive sector expands its own sector, the export skilled intensive sector, and education production, but contracts the import unskilled intensive sector. Above all, a technology improvement in the capital intensive non-traded sector raises wage differential in the short run and reduces the wage differential in the long run.

4.4 A Shock of Government Education Investment

Education capital and production jump. Since the government has to increase the tax rate to finance extra spending, the household is directly affected by a reduction in motivation to work and reduced consumption of goods. In the short run, the household cuts its service as skilled and unskilled labor and increases leisure because of the higher tax on working. This induces a lower education demand and a drop in the education price. In the long run, due to the increased supply and cheaper price, the household takes more education to form more skilled labor, pushing down the skilled wage. In the short run, the price of good 1 drops, contracting production in sector 1. Exports are boosted and imports pushed down. Sector 2 expands due to higher government demand and the cut in imports. The non-traded Sector 3 also expands to cope with the increased government demand. In the long run, the extra amount of skilled labor distributes into all three sectors expanding production. In the short run, the skilled intensive sector 1 and the government release skilled labor into the other two sectors, pushing down the skilled wage for the relatively abundant skilled labor. Since sector 2 is an unskilled intensive sector, and sector 3 also demands a certain amount of unskilled labor, this boosts the unskilled wage after the shock. The skilled wage bounces back, and then keeps declining below the base line into another steady state due to a skilled labor shortage in the short run and a glut in the long run. The unskilled wage drops after the first period and then increases to a higher level, due to the abundance of unskilled labor in the short run and a shortage in the long run. The picture of the variations of labor and wages is as follows: the government taxes both the skilled and unskilled labor income at the same rate and pulls this resource to education production. This represents taxing unskilled labor to subsidize skilled workers. The economy finally ends up with a glut of skilled labor and inadequate unskilled labor, which reverses the wage relativity. Leisure drops due to the substitution effect, which comes from a decrease in the household's disposable income.

To summarize the change of sectors and policy implications, an increase in government education investment expands all sectors. However, the household is disadvantaged by contracting goods consumption, due to a crowding out effect from the government spending. This case shows that government education investment has been excessive to its optimal level and creates a large distortionary effect which dominates the benefit of more education capacity. The government has to make a choice between approaching wage equality and disadvantaging the household. Generally speaking, an increase in government education investment converges the wage differential in the short run as well as in the long run[22]. A minor wage differential appears only for a short time period in the early stage.

4.5 A Shock of Foreign Income

A boost in exports make more imports affordable. The increasing international demand raises the price, production, and input hiring of good 1. The import sector contracts and employs less input, due to the influx of more foreign goods. In the short run, the expansion of the export sector also contracts the non-

[22] This result is consistent with the finding of Turrini (1998).

traded sector, because labor, especially skilled labor, is extracted from the non-traded and import sector. The contraction of sector 3 increases the price of good 3, and the extra demand for skilled and unskilled labor from sector 1 boosts both the skilled and unskilled wage. As sector 1 is skilled intensive, the skilled wage jumps more than the unskilled wage. This induces the household to invest more on education and boosts the education price. In the long run, due to more skilled labor being formed, the skilled wage declines but remains above the original level. The unskilled wage decreases below the baseline, due to the lower demand for unskilled labor. To meet the increased education demand, the government hires more skilled labor. The variation of government goods buying depends on the changes of goods price and market clearing conditions, which ends with buying more of good 2, and less of goods 1 and 3. The higher education price makes it feasible for government to cut the rate of taxation. In the short run, the household optimizes utility by consuming more of good 2 and less of goods 1 and 3. In the long run, the household increases consumption of goods 2 and 3, and decreases consumption of good 1. Due to the income effect, leisure increases. With the exception of sector 2, the other sectors accumulate more capital investment, so that aggregate capital in the economy jumps, leading to interest rate a drop in the short run.

To summarize the change of sectors and policy implications, an increase in foreign income or exports expands the export sector, the non-traded sector, and education production, but contracts the import sector. The household benefits from this shock by consuming more of goods 2, and 3, and by taking more leisure. Above all, a foreign income shock or an export shock enlarges wage differential to a large degree in the short run. In the long run, the wage differential is sustained, but to a small extent relative to the short run response.

Table 3 Simulation Results of Steady State with Shocks

Unit: %

Variables	A_{Q1}	A_{Q2}	A_{Q3}	I_E^G	Y^*
Q_1	14.0772	-0.4443	0.0708	0.1793	0.7526
J_1	14.0772	-0.4443	0.0708	0.1793	0.7526
I_1	14.0772	-0.4443	0.0708	0.1793	0.7526
$L_{s,1}^F$	-0.4354	-0.4354	0.0768	0.3971	0.7455
$L_{u,1}$	-0.4664	-0.4664	0.0559	-0.3633	0.7702
λ_1	0	0	0	0	0
K_1	14.0772	-0.4443	0.0708	0.1793	0.7526
Q_2	0.8873	15.603	-0.0136	0.0018	-1.3632
J_2	0.8873	15.603	-0.0136	0.0018	-1.3632
I_2	0.8873	15.603	-0.0136	0.0018	-1.3632
$L_{s,2}^F$	0.9098	0.9098	0.0013	0.5463	-1.3805
$L_{u,2}$	0.8783	0.8783	-0.0196	-0.2152	-1.3563
λ_2	0	0	0	0	0
K_2	0.8873	15.603	-0.0136	0.0018	-1.3632
Q_3	-0.0054	-0.0054	20.9928	0.2211	0.0018
J_3	-0.0054	-0.0054	20.9928	0.2211	0.0018
I_3	-0.0054	-0.0054	20.9928	0.2211	0.0018
$L_{s,3}^F$	0.0071	0.0071	0.0024	0.5263	-0.008
$L_{u,3}$	-0.0241	-0.0241	-0.0185	-0.235	0.0165
λ_3	0	0	0	0	0
K_3	-0.0054	-0.0054	20.9928	0.2211	0.0018

r	0	0	0	0	0
F	0.0044	14.5913	0.0147	-0.0614	0.0095
$I_{E,1}^{G}$	9.5081	-4.4317	-6.1528	10.1514	-0.0044
$I_{E,2}^{G}$	-4.4445	9.4934	-6.1612	9.7931	0.0061
$I_{E,3}^{G}$	-4.4351	-4.4351	13.5524	10.0558	-0.0016
P_{E}^{G}	-4.4445	9.4934	-6.1612	-0.1881	0.0061
J_s	-0.1437	-0.1437	0.0483	0.3439	0.2712
S_E	-0.1437	-0.1437	0.0483	0.3439	0.2712
P_E	-0.166	14.3961	0.0333	-9.2723	0.2888
L_s	-0.1437	-0.1437	0.0483	0.3439	0.2712
L_u	0.2352	0.2352	0.0053	-0.2679	-0.3517
l	-0.0438	-0.0438	-0.0494	-0.1197	0.0124
P_2	0	0	0	0	0
P_3	-0.0098	14.575	-17.3608	-0.2386	0.0077
W_u	0.0089	14.5964	0.006	0.2174	-0.007
W_s	-0.0223	14.5607	-0.0149	-0.5416	0.0175
W_s/W_u	-0.0312	-0.0312	-0.0209	-0.7574	0.0245
C_1	14.7869	0.1751	0.2421	-0.0849	-0.0009
C_2	0.1617	14.7715	0.2331	-0.4099	0.0096
C_3	0.1715	0.1715	21.29	-0.1717	0.0019
μ_1	4.1114	-10.3702	3.8022	0.2331	-0.0039
μ_2	3.9386	2.533	3.8368	-9.0608	0.2849
L_{s}^{G}	-0.2873	-0.2873	0.0965	-8.4646	0.5432
τ	-0.1896	-0.1896	-0.2667	0.4885	-0.0041
K_E	0	0	0	10	0
X	7.0521	-6.5751	0.0045	0.163	9.9942
M	-6.5875	7.0378	-0.0045	-0.1628	10.0058
P_1	-12.7411	14.5709	-0.009	-0.3253	0.0105
W_s/P_{E}^{G}	4.6279	4.6279	6.5498	-0.3541	0.0114
W_u/P_{E}^{G}	4.6606	4.6605	6.5721	0.4063	-0.0131

5 Conclusion

This paper discusses the relationship between productivity growth, trade, education, and the wage differential by constructing a DIGE open economy model. The main insight of this model is the choice of the household to undertake skill formation. This varies the endowment of skilled labor from short to long run. The result provides a comparison to a standard result from the Heckscher-Ohlin model in which skill endowments are fixed. Education is shown to be quite important yet its impingement on wage differential, is seldom discussed in the literature. It would be biased to only focus on the external trade effect in this issue. At the same time, an internal education policy should be recruited into the debate. The education issue is especially important when a developing country is investigated, due to the scarcity of skilled labor relative to a developed country. It is crucial to embed the dynamic adjustment process for guiding future econometric modeling in which data is used from a dynamic adjustment period. This model also shows the steady state relation which underlies the literature, can be misleading in the short run.

The main results from this model are that, in the long run, productivity growth and an increase in government education investment encourage wages to converge. At best productivity growth raises wage differential in the short run. An increase in trade creates wage differential. To summarize the results and policy implications from thirty different cases[23], wages tend to converge with a productivity upgrade[24]. It is also found that if the export sector is skilled intensive, an increase in export boosts wage differential; if the export sector is unskilled intensive, an export expansion reduces wage differential; if the export sector is capital intensive, an increase in exports may either boost or reduce wage differential. This small open economy model shows that a developed country's trade with a developing country causes wage differential in the developed country and wage convergence in the developing country. The result also tends to substantiate the argument that trade has a responsibility for wage differential. From a theoretical perspective it is unclear how productivity growth raises wage differential in the long run.

The contribution of the model in this paper is to enable the exploration of wage differential in both the short and long run. A couple of extensions of this model can be undertaken in future research to explore other interesting issues. For example, the effect of tariffs can be included. A test of the impact of unskilled labor immigration would be important for some countries. The model could also be extended to the case of an open capital account which may have important implications for the adjustment process to a range of shocks.

[23] Refer to thesis for a detailed discussion.

[24] The only two exceptions are the cases of unskilled sector 1, skilled sector 2, and capital intensive sector 3 with a larger ξ after a productivity shock in both the unskilled and skilled intensive sector, which end up with a minor wage differential.

Appendix

The parameters and exogenous variables settings are as follows:

$\alpha_1 = 0.3$;	$\beta_1 = 0.5$;	$\delta_1 = 0.1$;	$\phi_1 = 5$;
$\alpha_2 = 0.3$;	$\beta_2 = 0.2$;	$\delta_2 = 0.1$;	$\phi_2 = 5$;
$\alpha_3 = 0.5$;	$\beta_3 = 0.3$;	$\delta_3 = 0.1$;	$\phi_3 = 5$;
$\delta_s = 0.05$;	$\gamma_1 = 0.3$;	$\gamma_2 = 0.2$;	$\gamma_3 = 0.2$;
$\theta = 0.1$;	$\xi = 0.5$;	$\delta_E = 0.1$;	$\Phi = 10$;
$\varepsilon_1 = 1/3$;	$\varepsilon_2 = 1/3$;	$\rho = 0.5$;	$T = 8760$;
$A_{Qi} = 1$;	$c_{ji} = 1$;	$I_E^G = 100$;	$Y^* = 100$;

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