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CAMA Working Paper 65/2017

Revised version of working paper 23/2017

October 2017

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Keywords

Coskewness, Cokurtosis, Covolatility, Lagrange multiplier tests, European financial crisis, equity markets.

JEL Classification

C1, F3

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ISSN 2206-0332

Joint Tests of Contagion with Applications to Financial Crises*

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*The authors gratefully acknowledge ARC Discovery Project DP160102350 and DP120103443 funding. We also thank Joshua Chan, Thomas Flavin, Xun Lu, James Morley, Pengfei Wang, Benjamin Wong, Sen Xue, and participants of the brownbag seminar at Hong Kong University of Science and Technology, the National Taiwan University, the 20^{th} International Conference of Computing in Economics and Finance, and the Australasian Meeting of the Econometric Society. Author email addresses are: renee.mckibbin@anu.edu.au, ylhsiao@must.edu.mo, vance@unimelb.edu.au

1 Introduction

Tests of contagion are designed to identify the presence of additional factors that operate solely during financial crises which have the effect of increasing the connectedness of financial asset markets; see Dungey, Fry, Gonzalez-Hermosillo and Martin (2005) for a review of modelling approaches to contagion. Many of the existing tests of contagion proposed in the literature focus on either correlations (Forbes and Rigobon (2002)), coskewness (Fry, Martin and Tang (2010)), or on cokurtosis and covolatility (Fry-McKibbin and Hsiao (2016)). Correlations focus on the interaction between the expected returns in financial markets, whereas coskewness focusses on the interaction between the expected return and volatility in markets. Contagion through the cokurtosis channel measures the interaction effects between expected returns and skewness in financial markets, whereas covolatility focusses on contagion from volatility spillovers. An important feature of these tests is that they only consider testing specific contagion channels separately and do not provide an omnibus test of the overall importance of contagion.

Fry-McKibbin and Hsiao (2016) synthesize reasons for changes in the features of the distributions of asset returns in crisis periods. Mean-variance optimization of financial portfolios implies generally high excess returns in exchange for high risk (Sharpe (1964); Lintner (1965); Black (1972)). However, the regularly used assumption that the mean and variance are the only relevant moments for portfolio allocation misses important dynamics due to the asymmetry and (fat) tails of return distributions. The theoretical literature shows the reasons that higher order moments are important, particularly in crisis periods or periods of regime change. These channels include incomplete information and information asymmetries (Vaugirard (2007); Gârleanu et al. (2015)), liquidity and leverage constraints (Allen and Gale (2000); Cifuentes et al., (2005); Brunnermeier and Pedersen (2009); Caccioli et al. (2014)), safe havens (Vayanos, (2004)), wealth effects (Kyle and Xiong (2001)) and the (changing) risk preferences of investors (Kraus and Litzenberger (1976); Harvey and Siddique (2000); Fry, Martin and Tang (2010)).

A common feature of existing tests of contagion is that they focus on a single channel and do not necessarily consider the possibility of contagion operating through multiple channels. In contrast to these previous approaches the aim of this paper is to propose joint tests of contagion that allow for a range of contagious channels simultaneously. The approach is to construct Lagrange multiplier tests which are based on the likelihood associated with the multivariate generalized normal distribution of Fry, Martin and Tang (2010); see also Cobb, Koppstein and Chen (1983) and Lye and Martin (1993) for a discussion of the properties of univariate generalized exponential distributions. Working with this class of distributions provides a convenient framework as the role of higher order moments including coskewness, cokurtosis and covolatility are explicitly included in the form of the joint distribution. The finite sample properties of the joint contagion tests are examined using a range of Monte Carlo experiments with the sampling properties compared with a number of single equation contagion tests proposed in the literature.

The joint contagion testing framework also shares a broader relationship with the joint tests of multivariate normality originally proposed by Mardia (1970) and Bera and John (1983) and extended by Doornik and Hansen (2008), Zhou and Shao (2014) and Kim (2016). This earlier work tends to focus on joint multivariate tests of skewness and kurtosis combined with covolatility. However, this literature does not focus on testing for changes in these higher order moments which is desired in testing for contagion and which is the focus of the present paper.

There exists alternative, but related approaches to test for contagion in asset markets encompassing higher order distributional moments. The most relevant examples are copulas designed to test for comovements in the tails of the joint distribution and related to a joint test of coskewness, cokurtosis, or both (Rodriguez (2007); Busetti and Harvey (2011); Garcia and Tsafack (2011); Kenourgios, Samitas, and Paltalidis (2011)), and GARCH models which allow for time varying changes in the variance relates to testing for covolatility (Billio and Caporin (2005); Dungey, Milunovich, Thorp and Yang (2015)). Other related methods include asset price jumps and their spillovers (Grothe, Korniichuk, and Manner (2014) and Aït-Sahalia, Cacho-Diaz and Laeven (2015)), coexceedance tests for contagion (Favero and Giavazzi (2002), Bae et al. (2003), Pesaran and Pick (2007)), factor models of contagion (Dungey et al. (2010); Bekaert et al. (2014)), Markovian switching models (Gravelle, Kichian and Morley (2006); Rotta and Pereira (2016)) and wavelet analysis (Gallegati (2012)).

The new joint tests are applied to studying global and regional contagion in Eurozone equity markets during three financial crises: the subprime crisis in 2007-08, the global financial crisis (GFC) in 2008-09 and the European debt crisis from 2010-14. Using daily equity returns the empirical results highlight the importance of higher order moment channels in transmitting contagion across equity markets globally as well as regionally. The empirical results also show that for some countries traditional measures of contagion based on correlations can fail to detect contagion when it is present in higher order moments.

The rest of the paper proceeds as follows. Section 2 provides the main framework for constructing joint tests of contagion. The finite sample properties of the tests are then presented in Section 3 using a range of Monte Carlo experiments. Both size and power properties of the joint tests are presented which are compared with a number of existing tests proposed in the literature that focus on single channels of contagion. The proposed tests are applied in Section 4 to study the presence of contagion in the Eurozone during three financial crises. Concluding comments are given in Section 5. All derivations are given in Appendix A.

2 Testing Framework

This section provides the main framework for constructing joint tests of contagion. Consider the bivariate generalized exponential distribution proposed by Fry, Martin and Tang (2010) for the two random variables r_i and r_j

$$f(r_{it}, r_{jt}) = \exp\left(h_t - \eta_t\right), \tag{1}$$

where h_t is specified as

$$\begin{aligned} h_t \;=\; & -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2 + \left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 - 2\rho\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)\right) \\ & +\theta_4\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 + \theta_5\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1 \\ & +\theta_6\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^3 + \theta_7\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^3\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1 \\ & +\theta_8\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2, \end{aligned} \tag{2}$$

and η is the normalizing constant

$$\eta_t = \ln\int\int \exp\left(h_t\right) dr_i dr_j. \tag{3}$$

The distribution in (1) is an extension of the univariate generalized distribution discussed by Cobb, Koppstein and Chen (1983) and Lye and Martin (1993). The choice of h_t represents the generalized bivariate normal distribution which is a subordinate distribution of the generalized exponential class. A variant of this distribution is the bivariate generalized lognormal distribution which is used by Fry-McKibbin, Martin and Tang (2014) to study the effects of pricing options during periods of financial stress. The parameters θ_4 to θ_8 control departures from bivariate normality with θ_4 and θ_5 controlling coskewness, θ_6 and θ_7 controlling cokurtosis and θ_8 controlling covolatility. In the special case where $\theta_4 = \theta_5 = \theta_6 = \theta_7 = \theta_8 = 0$ in (2), the distribution in (1 reduces to the bivariate normal distribution. In applying the framework to testing for contagion r_i will represent the returns on equities in the i^{th} country.

For a sample of $t = 1, 2, \cdots, T$ observations the log-likelihood corresponding to (1) is of the form

$$\ln L(\Theta) = \frac{1}{T}\sum_{t=1}^{T} h_t(\Theta) - \frac{1}{T}\sum_{t=1}^{T} \eta_t(\Theta), \tag{4}$$

where $\Theta = \left\{\mu_i, \mu_j, \sigma_i^2, \sigma_j^2, \rho, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\right\}$ is a vector of the unknown parameters. A convenient testing framework is to use the Lagrange multiplier test as given by

$$LM = TG(\widehat{\Theta})' I(\widehat{\Theta})^{-1} G(\widehat{\Theta}), \tag{5}$$

where $\widehat{\Theta}$ represent the maximum likelihood estimator of Θ under the null hypothesis, $G(\widehat{\Theta})$ is the score function evaluated at $\widehat{\Theta}$ given by

$$G(\widehat{\Theta}) = \left(\frac{\partial \ln L(\Theta)}{\partial \Theta}\right)\bigg|_{\Theta=\widehat{\Theta}}, \tag{6}$$

and $I(\widehat{\Theta})$ is the asymptotic information matrix evaluated at $\widehat{\Theta}$. Fry, Martin and Tang (2010) show that the information matrix $I(\theta)$ has a particularly convenient form given by

$$I(\Theta) = E\left[\frac{\partial h_t}{\partial \Theta}\frac{\partial h_t}{\partial \Theta'}\right] - E\left[\frac{\partial h_t}{\partial \Theta}\right] E\left[\frac{\partial h_t}{\partial \Theta'}\right]. \tag{7}$$

The advantage of this result is that it simplifies the construction of Lagrange multiplier tests of higher order comoments as the expectations in (7) are evaluated under the null hypothesis. A further advantage of constructing tests of higher comoments this way is that for certain distributional specifications of (2) the form of the Lagrange multiplier statistic is equivalent to existing comoment tests adopted in the literature by Fry, Martin and Tang (2010), and Fry-McKibbin and Hsiao (2016) for example.

The Lagrange multiplier test statistic in (5) is asymptotically distributed under the null hypothesis as χ_p^2, where p represents the number of restrictions imposed on the model. Alternatively, a robust version of the Lagrange multiplier test in (5) is obtained by replacing the information matrix $I(\widehat{\Theta})$ by the corresponding quasi maximum likelihood covariance estimator $H(\widehat{\Theta})J(\widehat{\Theta})^{-1}H(\widehat{\Theta})$ where $H(\widehat{\Theta})$ is the Hessian and $J(\widehat{\Theta})$ is the outer product of the gradient matrix computed as

$$J(\widehat{\Theta}) = \frac{1}{T}\sum_{t=1}^{T} g_t(\widehat{\Theta})g_t'(\widehat{\Theta}), \tag{8}$$

where $g_t = \partial \ln L_t(\Theta) / \partial\Theta = \partial h_t/\partial\Theta - \partial\eta_t/\partial\Theta$ is the gradient vector at time t.

In developing the tests of contagion the following notation is used. The first period corresponds to the noncrisis period which is denoted as x, while the second period corresponds to the crisis period and is denoted as y. The sample sizes of the two periods are T_x and T_y respectively. The correlation between the two asset returns is denoted as ρ_x (noncrisis) and ρ_y (crisis). The estimated parameters $\widehat{\mu}_{ix}$, $\widehat{\mu}_{jx}$, $\widehat{\mu}_{iy}$ and $\widehat{\mu}_{jy}$, are the sample means of the asset returns for markets i and j during the periods x and y respectively, and $\widehat{\sigma}_{ix}$, $\widehat{\sigma}_{jx}$, $\widehat{\sigma}_{iy}$ and $\widehat{\sigma}_{jy}$ are the corresponding sample standard deviations.

2.1 Joint Contagion Test

The first joint contagion test presented is the most general test statistic as it is designed to identify contagion through changes in coskewness, cokurtosis and covolatility. Let coskewness in the noncrisis (x) and crisis (y) periods be

$$\widehat{\psi}_k(r_i^2, r_j^1) = \frac{1}{T_k}\sum_{t=1}^{T_k}\left(\frac{r_{it}-\widehat{\mu}_{ik}}{\widehat{\sigma}_{ik}}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_{jk}}{\widehat{\sigma}_{jk}}\right)^1, \quad k = x, y \tag{9}$$

$$\widehat{\psi}_k(r_i^1, r_j^2) = \frac{1}{T_k}\sum_{t=1}^{T_k}\left(\frac{r_{it}-\widehat{\mu}_{ik}}{\widehat{\sigma}_{ik}}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_{jk}}{\widehat{\sigma}_{jk}}\right)^2, \quad k = x, y, \tag{10}$$

and the corresponding cokurtosis moments be

$$\widehat{\xi}_k(r_i^3, r_j^1) = \frac{1}{T_k}\sum_{t=1}^{T_k}\left(\frac{r_{it}-\widehat{\mu}_{ik}}{\widehat{\sigma}_{ik}}\right)^3\left(\frac{r_{jt}-\widehat{\mu}_{jk}}{\widehat{\sigma}_{jk}}\right)^1, \quad k = x, y \tag{11}$$

$$\widehat{\xi}_k(r_i^1, r_j^3) = \frac{1}{T_k}\sum_{t=1}^{T_k}\left(\frac{r_{it}-\widehat{\mu}_{ik}}{\widehat{\sigma}_{ik}}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_{jk}}{\widehat{\sigma}_{jk}}\right)^3, \quad k = x, y. \tag{12}$$

Finally, the covolatility moments are defined as

$$\widehat{\varphi}_k(r_i^2, r_j^2) = \frac{1}{T_k}\sum_{t=1}^{T_k}\left(\frac{r_{it}-\widehat{\mu}_{ik}}{\widehat{\sigma}_{ik}}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_{jk}}{\widehat{\sigma}_{jk}}\right)^2, \quad k = x, y. \tag{13}$$

The joint contagion test statistic is defined in terms of the changes in the comoments as given in equations (9) to (13) between the crisis and noncrisis periods. This statistic consists of six components with the first five components corresponding to the five excess comoments being tested and the sixth capturing the interaction effects between the pertinent excess comoments. The form of the statistic is (see Appendix A for details)

$$JOINT = J_{12} + J_{21} + J_{13} + J_{31} + J_{22} + J_I, \tag{14}$$

where

$$J_{12} = \frac{(\widehat{\psi}_y(r_i^1, r_j^2) - \widehat{\psi}_x(r_i^1, r_j^2))^2}{\frac{2(1-\widehat{\upsilon}^6_{y|x_i})}{T_y(2\widehat{\upsilon}^2_{y|x_i}+1)} + \frac{2(1-\widehat{\rho}^6_x)}{T_x(2\widehat{\rho}^2_x+1)}}, \qquad J_{21} = \frac{(\widehat{\psi}_y(r_i^2, r_j^1) - \widehat{\psi}_x(r_i^2, r_j^1))^2}{\frac{2(1-\widehat{\upsilon}^6_{y|x_i})}{T_y(2\widehat{\upsilon}^2_{y|x_i}+1)} + \frac{2(1-\widehat{\rho}^6_x)}{T_x(2\widehat{\rho}^2_x+1)}},$$

are the excess coskewness components,

$$\begin{aligned} J_{13} &= \frac{(\widehat{\xi}_y(r_i^1, r_j^3) - \widehat{\xi}_x(r_i^1, r_j^3))^2}{\frac{6(\widehat{\upsilon}^{10}_{y|x_i} - \widehat{\upsilon}^8_{y|x_i} - \widehat{\upsilon}^2_{y|x_i} + 1)}{T_y(3\widehat{\upsilon}^4_{y|x_i} + 2\widehat{\upsilon}^2_{y|x_i} + 1)} + \frac{6(\widehat{\rho}^{10}_x - \widehat{\rho}^8_x - \widehat{\rho}^2_x + 1)}{T_x(3\widehat{\rho}^4_x + 2\widehat{\rho}^2_x + 1)}}, \\ J_{31} &= \frac{(\widehat{\xi}_y(r_i^3, r_j^1) - \widehat{\xi}_x(r_i^3, r_j^1))^2}{\frac{6(\widehat{\upsilon}^{10}_{y|x_i} - \widehat{\upsilon}^8_{y|x_i} - \widehat{\upsilon}^2_{y|x_i} + 1)}{T_y(3\widehat{\upsilon}^4_{y|x_i} + 2\widehat{\upsilon}^2_{y|x_i} + 1)} + \frac{6(\widehat{\rho}^{10}_x - \widehat{\rho}^8_x - \widehat{\rho}^2_x + 1)}{T_x(3\widehat{\rho}^4_x + 2\widehat{\rho}^2_x + 1)}}, \end{aligned}$$

are the excess cokurtosis components and

$$J_{22} = \frac{(\widehat{\varphi}_y(r_i^2, r_j^2) - \widehat{\varphi}_x(r_i^2, r_j^2))}{\frac{4(\widehat{\upsilon}^2_{y|x_i} - 1)^2(\widehat{\upsilon}^4_{y|x_i} + 1)}{T_y(\widehat{\upsilon}^4_{y|x_i} + 6\widehat{\upsilon}^2_{y|x_i} + 1)} + \frac{4(\widehat{\rho}^2_x - 1)^2(\widehat{\rho}^4_x + 1)}{T_x(\widehat{\rho}^4_x + 6\widehat{\rho}^2_x + 1)}},$$

in the excess covolatility component. The last term in (14) captures the interaction effects between the excess higher order moments

$$
\begin{aligned}
J_I &= -\frac{(\widehat{\psi}_y(r_i^1, r_j^2) - \widehat{\psi}_x(r_i^1, r_j^2))(\widehat{\psi}_y(r_i^2, r_j^1) - \widehat{\psi}_x(r_i^2, r_j^1))}{\frac{1-\widehat{\upsilon}_{y|x_i}^6}{T_y(\widehat{\upsilon}_{y|x_i}^3 + 2\widehat{\upsilon}_{y|x_i})} + \frac{1-\widehat{\rho}_x^6}{T_x(\widehat{\rho}_x^3 + 2\widehat{\rho}_x)}} \\
&\quad + \frac{(\widehat{\xi}_y(r_i^1, r_j^3) - \widehat{\xi}_x(r_i^1, r_j^3))(\widehat{\xi}_y(r_i^3, r_j^1) - \widehat{\xi}_x(r_i^3, r_j^1))}{\frac{3(\widehat{\upsilon}_{y|x_i}^{10} - \widehat{\upsilon}_{y|x_i}^8 - \widehat{\upsilon}_{y|x_i}^2 + 1)}{T_y(\widehat{\upsilon}_{y|x_i}^6 + 2\widehat{\upsilon}_{y|x_i}^4 + 3\widehat{\upsilon}_{y|x_i}^2)} + \frac{3(\widehat{\rho}_x^{10} - \widehat{\rho}_x^8 - \widehat{\rho}_x^2 + 1)}{T_x(\widehat{\rho}_x^6 + 2\widehat{\rho}_x^4 + 3\widehat{\rho}_x^2)}} \\
&\quad - \frac{(\widehat{\xi}_y(r_i^1, r_j^3) - \widehat{\xi}_x(r_i^1, r_j^3))(\widehat{\varphi}_y(r_i^2, r_j^2) - \widehat{\varphi}_x(r_i^2, r_j^2))}{\frac{(\widehat{\upsilon}_{y|x_i}^2 - 1)^2(\widehat{\upsilon}_{y|x_i}^4 + 1)}{T_y(\widehat{\upsilon}_{y|x_i}^3 + \widehat{\upsilon}_{y|x_i})} + \frac{(\widehat{\rho}_x^2 - 1)^2(\widehat{\rho}_x^4 + 1)}{T_x(\widehat{\rho}_x^3 + \widehat{\rho}_x)}} \\
&\quad - \frac{(\widehat{\xi}_y(r_i^3, r_j^1) - \widehat{\xi}_x(r_i^3, r_j^1))(\widehat{\varphi}_y(r_i^2, r_j^2) - \widehat{\varphi}_x(r_i^2, r_j^2))}{\frac{(\widehat{\upsilon}_{y|x_i}^2 - 1)^2(\widehat{\upsilon}_{y|x_i}^4 + 1)}{T_y(\widehat{\upsilon}_{y|x_i}^3 + \widehat{\upsilon}_{y|x_i})} + \frac{(\widehat{\rho}_x^2 - 1)^2(\widehat{\rho}_x^4 + 1)}{T_x(\widehat{\rho}_x^3 + \widehat{\rho}_x)}}.
\end{aligned}
$$

The term $\widehat{\upsilon}_{y|x_i}$ in the above expressions is the Forbes and Rigobon (2002) heteroskedastic adjusted correlation coefficient given by

$$
\widehat{\upsilon}_{y|x_i} = \frac{\widehat{\rho}_y}{\sqrt{1 + \left(\frac{s_{iy}^2 - s_{ix}^2}{s_{ix}^2}\right)(1 - \widehat{\rho}_y^2)}}. \tag{15}
$$

Under the null hypothesis of no contagion, the test statistic in (14) is asymptotically distributed as $JOINT \xrightarrow{d} \chi_5^2$, where the number of degrees of freedom is determined by the number of restrictions imposed on (2) under the null hypothesis which for this class of tests is $\theta_4 = \theta_5 = \theta_6 = \theta_7 = \theta_8 = 0$.

2.2 Joint Coskewness Contagion Test

The next proposed test is a restricted version of the joint test statistic $JOINT$ in (14) where the transmission channels of contagion solely arise from changes in coskewness

(see Appendix A for details)

$$COSKEW = \frac{(\widehat{\psi}_y(r_i^1, r_j^2) - \widehat{\psi}_x(r_i^1, r_j^2))^2 + (\widehat{\psi}_y(r_i^2, r_j^1) - \widehat{\psi}_x(r_i^2, r_j^1))^2}{\dfrac{2(1-\widehat{v}^6_{y|x_i})}{T_y(2\widehat{v}^2_{y|x_i}+1)} + \dfrac{2(1-\widehat{\rho}^6_x)}{T_x(2\widehat{\rho}^2_x+1)}} - \frac{(\widehat{\psi}_y(r_i^1, r_j^2) - \widehat{\psi}_x(r_i^1, r_j^2))(\widehat{\psi}_y(r_i^2, r_j^1) - \widehat{\psi}_x(r_i^2, r_j^1))}{\dfrac{\left((1-\widehat{v}^6_{y|x_i})\right)}{T_y(\widehat{v}^3_{y|x_i}+2\widehat{v}_{y|x_i})} + \dfrac{(1-\widehat{\rho}^6_x)}{T_x(\widehat{\rho}^3_x+2\widehat{\rho}_x)}}. \tag{16}$$

The first two components represent the types of coskewness being tested and the third component captures the interaction effect between the two coskewness comoments.[1] Under the null hypothesis this statistic is asymptotically distributed as $COSKEW \xrightarrow{d} \chi^2_2$.

2.3 Comparison with Existing Tests

The joint tests given in (14) and (16) are by construction tests of several potential transmission channels of contagion operating simultaneously. In contrast, many of the existing tests of contagion adopted in the literature tend to focus on individual contagion channels.

The Fry, Martin and Tang (2010) contagion test focuses on changes in coskewness between the crisis and noncrisis periods given by

$$CS_{21} = \frac{(\widehat{\psi}_y(r_i^2, r_j^1) - \widehat{\psi}_x(r_i^2, r_j^1))^2}{\dfrac{4\widehat{\nu}^2_{y|x_i}+2}{T_y} + \dfrac{4\widehat{\rho}^2_x+2}{T_x}}, \tag{17}$$

where $\widehat{\psi}_k(r_i^2, r_j^1)$ $k = x, y$ is defined in (9). Under the null hypothesis CS_{21} is asymptotically distributed as $CS_{21} \xrightarrow{d} \chi^2_1$. Reversing the order of the components of the statistic in (17) so as to test for coskewness between r_i and r_j^2 yields a second version of the coskewness test denoted as CS_{12}, which has the same asymptotic distribution.

[1] A natural extension of the proposed joint tests is to construct a test focussing on the even-ordered moments of cokurtosis and covolatility. This test is not constructed here but could be done following the strategy of the joint test $COSKEW$. Instead, the approach adopted here is to interpret jointly the two joint tests $JOINT$ and $COSKEW$ and infer the relative role of higher order even moment channels. This strategy is further complemented by also interpreting the properties of the joint tests together with the single contagion channel tests as well.

Instead of focussing on coskewness, Fry-McKibbin and Hsiao (2016) base their contagion test on a change in cokurtosis

$$CK_{31} = \frac{(\widehat{\xi}_y(r_i^3, r_j^1) - \widehat{\xi}_x(r_i^3, r_j^1))^2}{\dfrac{18\widehat{\upsilon}_{y|x_i}^2 + 6}{T_y} + \dfrac{18\widehat{\rho}_x^2 + 6}{T_x}}, \tag{18}$$

where $\widehat{\xi}_k(r_i^3, r_j^1)$ $k = x, y$ is defined in (11). Under the null hypothesis CK_{31} is asymptotically distributed as $CK_{31} \xrightarrow{d} \chi_1^2$. As with the coskewness test in (17), the components can be reversed to test for cokurtosis between r_i and r_j^3, to produce a test statistic denoted as CK_{13} which has the same asymptotic distribution as CK_{31}.

Fry-McKibbin and Hsiao (2014) also propose a covolatility test of contagion which is designed to identify significant changes in covolatility during a financial crisis. The test is given by

$$CV_{22} = \frac{(\widehat{\varphi}_y(r_i^2, r_j^2) - \widehat{\varphi}_x(r_i^2, r_j^2))^2}{\dfrac{4\widehat{\upsilon}_{y|x_i}^4 + 16\widehat{\upsilon}_{y|x_i}^2 + 4}{T_y} + \dfrac{4\widehat{\rho}_x^4 + 16\widehat{\rho}_x^2 + 4}{T_x}}, \tag{19}$$

where $\widehat{\varphi}_k(r_i^2, r_j^2)$ $k = x, y$ is defined in (13). Under the null hypothesis the CV_{22} statistic is asymptotically distributed as $CV_{22} \xrightarrow{d} \chi_1^2$.

Finally the variant of the Forbes and Rigobon (2002) contagion test proposed by Fry, Martin and Tang (2010) is based on comparing the (adjusted) correlations between the crisis and noncrisis periods

$$FR = \left(\frac{\widehat{\upsilon}_{y|x_i} - \widehat{\rho}_x}{\sqrt{var\left(\widehat{\upsilon}_{y|x_i} - \widehat{\rho}_x\right)}} \right)^2, \tag{20}$$

where $\widehat{\upsilon}_{y|x_i}$ is the adjusted correlation coefficient in the crisis period as defined in (15). The variance in (20) is given by

$$Var\left(\widehat{\nu}_{y|x_i} - \widehat{\rho}_x\right) = Var\left(\widehat{\nu}_{y|x_{wi}}\right) + Var\left(\widehat{\rho}_x\right) - 2Cov\left(\widehat{\nu}_{y|x_i}, \widehat{\rho}_x\right),$$

where

$$\begin{aligned}
Var\left(\widehat{\nu}_{y|x_i}\right) &= \frac{1}{2}\frac{(1+\delta)^2}{\left[1+\delta\left(1-\rho_y^2\right)\right]^3}\left[\frac{1}{T_y}\left(\left(2-\rho_y^2\right)\left(1-\rho_y^2\right)^2\right) + \frac{1}{T_x}\left(\rho_y^2\left(1-\rho_y^2\right)^2\right)\right], \\
Var\left(\widehat{\rho}_x\right) &= \frac{1}{T_x}\left(1-\rho_x^2\right)^2, \\
Cov\left(\widehat{\nu}_{y|x_i}, \widehat{\rho}_x\right) &= \frac{1}{2}\frac{1}{T_x}\frac{\rho_y\rho_x\left(1-\rho_y^2\right)\left(1-\rho_x^2\right)(1+\delta)}{\sqrt{\left[1+\delta\left(1-\rho_y^2\right)\right]^3}},
\end{aligned}$$

and $\delta = (s_{iy}^2 - s_{ix}^2)/s_{ix}^2$ represents the proportionate change in the variance of r_i across the two sample periods, and s_{ix}^2 and s_{iy}^2 are the respective sample variances. Under the null hypothesis the FR statistic is asymptotically distributed as $FR \xrightarrow{d} \chi_1^2$.

3 Finite Sample Properties

The finite sample properties of the joint tests $JOINT$ in (14) and $COKEW$ in (16) are now investigated using a range of Monte Carlo experiments. For comparison, the finite sample properties of the single channel contagion tests proposed in the literature given by (17) to (20) are also investigated. The data generating process is based on the bivariate generalized normal distribution in (1) where the population means and variances are standardized at $\mu_i = \mu_j = 0$, and $\sigma_i^2 = \sigma_j^2 = 1$, respectively, yielding the joint density function

$$f(r_i, r_j) = \exp\left[-\left(\frac{r_i^2 + r_j^2 - 2\rho r_i r_j}{2(1-\rho^2)}\right) + \theta_4 r_i r_j^2 + \theta_5 r_i^2 r_j + \theta_6 r_i r_j^3 + \theta_7 r_i^3 r_j + \theta_8 r_i^2 r_j^2 - \eta\right]. \tag{21}$$

3.1 Size

The size properties of the contagion tests are conducted under the null hypothesis of no contagion by imposing the restrictions

$$\theta_4 = \theta_5 = \theta_6 = \theta_7 = \theta_8 = 0,$$

on the data generating process so (21) reduces to a bivariate normal distribution in the noncrisis and crisis periods. The sample sizes are set to $T_x = 500$ for the noncrisis period and vary over the range $T_y = \{50, 100, 200, 300, 400, 500\}$ for the crisis period. The number of replications in all simulation experiments is 50000.

The size properties of the joint contagion tests, $JOINT$ and $COSKEW$, are presented in Table 1 for the case where the correlation parameter in (21) is set at $\rho = 0$ in the noncrisis and crisis periods. The sizes for the joint tests are based on the 5% asymptotic χ_5^2 critical value in the case of $JOINT$ and the χ_2^2 critical value in the case of $COSKEW$. Also presented are the single channel contagion tests consisting of the covolatility test CV_{22} in (19), the cokurtosis test CK_{31} in (18), the coskewness test CS_{21} in (17) and the Forbes-Rigobon correlation test FR in (20). For these tests the sizes are based on the 5% asymptotic χ_1^2 critical value.

Table 1 shows that the joint test $JOINT$ is slightly over sized, but is very stable across all crisis sample sizes from small samples of $T_y = 50$ to larger samples of size $T_y = 500$. The coskewness joint test $COSKEW$ exhibits slightly better sizes than the $JOINT$ test as they are closer to the nominal size of 5%. As with the $JOINT$ test, the sizes of the $COSKEW$ test are also vary stable across all crisis sample sizes investigated. This discrepancy in the size of the two joint tests is largely the result of $COSKEW$ just being a function of third order moments, whereas $JOINT$ also requires fourth order moments which are relatively more difficult to estimate precisely than third order moments. There is also a further loss in precision with the $JOINT$ test over the $COSKEW$ test which is a reflection that the latter test is just based on testing for coskewness whereas the former test requires testing both coskewness and the fourth order comoments of cokurtosis and covolatility.

The single channel contagion tests CV_{22}, CK_{31} and CS_{21} in Table 1 tend to be undersized for crisis sample sizes of $T_y = 50$. The extent of these statistics being undersized is a function of the order of the moment being tested with CV_{22} being more undersized that CK_{31}, which in turn, is more undersized than CS_{21}. The simulation results show that all three tests are consistent with the empirical sizes approaching the nominal 5% level as the sample size for the crisis period increases.

The FR test in Table 1 is marginally oversized for crisis samples of size $T_y = 50$, where the empirical size is 0.071 compared to the nominal size of 5%. As with the single channel higher order moment tests, the size experiments show that the FR test is also a consistent test with its size quickly approaching the nominal size of 5% for increases in the crisis period T_y.

3.2 Power

The power properties of the contagion tests are now investigated where contagion occurs in the crisis period through the higher order moment channels. In all experiments the noncrisis and crisis sample sizes are set at $T_x = T_y = 500$. Four experiments are conducted. The first three represent cases where a single channel of contagion operates, whereas for the fourth experiment multiple channels are allowed to operate. In simulating the generalized bivariate normal distribution under the alternative hypothesis, as there is no analytical expression for the inverse of its cumulative distribution function the random variables are simulated by numerical evaluation of the inverse-transform.

Table 1:
Size properties of contagion tests based on the 5% nominal significance level. The sample size for the noncrisis period is $T_x = 500$ whereas for the crisis period it ranges over $T_y = \{50, 100, 200, 300, 400, 500\}$. The number of replications is 50000.

Statistic	Sample size in the crisis period (T_y)					
	50	100	200	300	400	500
$JOINT$	0.071	0.071	0.072	0.069	0.067	0.069
$COSKEW$	0.056	0.060	0.060	0.061	0.058	0.059
CV_{22}	0.029	0.040	0.046	0.047	0.048	0.046
CK_{31}	0.036	0.044	0.048	0.047	0.049	0.049
CS_{21}	0.043	0.048	0.048	0.051	0.048	0.051
FR	0.071	0.059	0.053	0.052	0.052	0.051

3.2.1 Experiment I

In the first experiment contagion in the crisis period is transmitted through the covolatility channel, with the generalized bivariate normal distribution specified as

$$f(r_i, r_j) = \exp\left[-\frac{r_i^2 + r_j^2}{2} + \theta_8 r_i^2 r_j^2 - \eta\right], \tag{22}$$

where the covolatility parameter θ_8 takes on the values

$$\theta_8 = \{0.0, -0.1, -0.2, -0.3, -0.4, -0.5, -0.6, -0.7, -0.8, -0.9\},$$

with increasing (absolute) values representing increasing covolatility in the crisis period. The noncrisis period is where $\theta_8 = 0.0$, corresponding to the standardized bivariate normal distribution with $\rho = 0$.

The results of this experiment for the joint and single channel contagion tests are given in the first block of Table 2 under the header Experiment I. The power of the tests are size adjusted so the power reported under the null hypothesis $\theta_8 = 0$, equals the nominal size value of 0.05. The joint test $JOINT$ displays increasing power as the covolatility parameter θ_8 in creases in absolute terms from 0.0 to -0.9. The single channel covolatility test CV_{22}, also shows increasing power and indeed exhibits higher power than the $JOINT$ test for this experiment. This is a reflection that the CV_{22} statistic is solely designed to test for contagion arising from covolatility whereas $JOINT$ by its very nature demonstrates some loss of power due to it being a joint test that is testing across several other contagious channels which are not all active in this experiment. Not surprisingly the joint and single channel coskewness tests $COSKEW$

and CS_{21} respectively, exhibit zero power as the experiment is about contagion from covolatility and not from coskewness. Interestingly, even though the cokurtosis test CK_{31} is a test about fourth order moments, it also exhibits zero power in identifying changes in contagion arising from covolatility.

As with the coskewness and cokurtosis tests, the FR test also lacks power in testing for contagion from covolatility. This observation is an important result as the FR statistic is commonly used as a test of contagion in empirical work and which commonly does not identify contagion; see for example, the empirical applications in Forbes and Rigobon (2002). This result would then imply that even if the FR test fails to identify contagion, that does not necessarily mean that contagion is not operating as there could be other active contagious channels operating at higher order moments.

3.2.2 Experiment II

The second experiment is based on the contagion channel operating during the crisis period through coskewness, with the generalized bivariate normal distribution specified as

$$f(r_i, r_j) = \exp\left[-\frac{r_i^2 + r_j^2}{2} + \theta_5 r_i^2 r_j - 0.5 r_i^2 r_j^2 - \eta\right], \tag{23}$$

where the coskewness parameter θ_5 takes on the values

$$\theta_5 = \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\},$$

with increasing values representing increasing coskewness in the crisis period. The noncrisis period is where there is no coskewness $\theta_5 = 0.0$, whilst the crisis period is where $\theta_5 > 0$.

The results of this experiment are given in the second block of Table 2 under the header Experiment II. Both joint contagion tests $JOINT$ and $COSKEW$ demonstrate monotonically increasing power functions. The $COSKEW$ joint test displays greater power than the $JOINT$ test as the channel of contagion operating is just coskewness in this experiment. In fact the single coskewness channel test CS_{21} exhibits even greater power than the joint coskewness test $COSKEW$ as the latter statistic is testing across both types of contagion channels, whereas only one of the two channels is operating in the experiment. As expected, the contagion single channel contagion tests based on even order moments, CV_{22}, CK_{31} and FR, exhibit no or very little power in identifying the coskewness channel. Again, as with experiment I, the widely used FR test does not have power in detecting higher order moment contagion.

Table 2:

Size adjusted power properties of contagion tests. The noncrisis and crisis sample sizes are $T_x = T_y = 500$. The number of replications is 50000.

Statistic					Experiment Type					
					Experiment I based on equation (22)					
$\theta_8 =$	0.00	-0.10	-0.20	-0.30	-0.40	-0.50	-0.60	-0.70	-0.80	-0.90
$JOINT$	0.05	0.13	0.29	0.42	0.55	0.68	0.76	0.82	0.83	0.90
$COSKEW$	0.05	0.02	0.02	0.01	0.02	0.02	0.02	0.02	0.02	0.02
CV_{22}	0.05	0.38	0.71	0.89	0.95	0.99	0.99	1.00	1.00	1.00
CK_{31}	0.05	0.02	0.02	0.03	0.02	0.02	0.03	0.02	0.03	0.03
CS_{21}	0.05	0.03	0.01	0.02	0.02	0.02	0.03	0.01	0.01	0.02
FR	0.05	0.05	0.04	0.05	0.04	0.04	0.02	0.02	0.03	0.02
					Experiment II based on equation (23)					
$\theta_5 =$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$JOINT$	0.05	0.04	0.04	0.04	0.06	0.10	0.14	0.31	0.53	0.83
$COSKEW$	0.05	0.07	0.12	0.23	0.38	0.58	0.77	0.92	0.98	0.99
CV_{22}	0.05	0.08	0.06	0.07	0.07	0.08	0.06	0.08	0.10	0.11
CK_{31}	0.05	0.06	0.05	0.06	0.05	0.05	0.07	0.06	0.07	0.06
CS_{21}	0.05	0.07	0.16	0.30	0.52	0.72	0.88	0.98	0.99	1.00
FR	0.05	0.04	0.07	0.05	0.06	0.06	0.06	0.05	0.06	0.07
					Experiment III based on equation (24)					
$\rho =$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$JOINT$	0.05	0.04	0.07	0.11	0.16	0.23	0.34	0.44	0.50	0.56
$COSKEW$	0.05	0.04	0.06	0.07	0.09	0.12	0.17	0.18	0.26	0.31
CV_{22}	0.05	0.05	0.05	0.05	0.06	0.07	0.06	0.07	0.05	0.05
CK_{31}	0.05	0.05	0.06	0.06	0.07	0.06	0.06	0.05	0.05	0.05
CS_{21}	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.04
FR	0.05	0.37	0.91	1.00	1.00	1.00	1.00	1.00	1.00	1.00
					Experiment IV based on equation (25)					
$\theta_8 =$	0.00	-0.10	-0.20	-0.30	-0.40	-0.50	-0.60	-0.70	-0.80	-0.90
$\rho =$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$JOINT$	0.05	0.14	0.41	0.71	0.91	0.99	1.00	1.00	1.00	1.00
$COSKEW$	0.05	0.03	0.03	0.02	0.03	0.03	0.04	0.05	0.07	0.13
CV_{22}	0.05	0.34	0.69	0.86	0.95	0.98	1.00	1.00	1.00	1.00
CK_{31}	0.05	0.04	0.09	0.19	0.38	0.65	0.89	0.99	1.00	1.00
CS_{21}	0.05	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01
FR	0.05	0.19	0.49	0.79	0.96	1.00	1.00	1.00	1.00	1.00

3.2.3 Experiment III

The third experiment allows for contagion to operate through the supposedly more traditional correlation channel. Under this scenario the FR test is expected to be more powerful as this is the type of situation that this test is designed for. The generalized bivariate normal distribution for the experiment is specified as

$$f(r_i, r_j) = \exp\left[-\left(\frac{r_i^2 + r_j^2 - 2\rho r_i r_j}{2(1-\rho^2)}\right) - 0.5 r_i^2 r_j^2 - \eta\right], \tag{24}$$

where the correlation parameter is chosen as

$$\rho = \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}.$$

The noncrisis period corresponds to $\rho = 0.0$, and the crisis period to $\rho > 0$.

The results of this experiment are given in third block of Table 2. As expected the FR test is the most powerful of all tests, including all joint and single channel tests, and reaches power of unity for $\rho = 0.3$. The joint tests $JOINT$ and $COSKEW$ nonetheless still exhibit some power in identifying the correlation channel even though these tests are primarily designed to test for contagion arising from third or fourth order moments. Effectively this power is coming about indirectly through the joint test statistics being a function of the correlation parameters: the Forbes-Rigobon adjusted correlation parameter in the crisis period and the unadjusted correlation parameter in the noncrisis period. Of the two joint tests, it is the more general version given by $JOINT$ which displays better power properties than the $COSKEW$ test. In contrast, the single channel higher order moment contagion tests CV_{22}, CK_{31} and CS_{21}, display no power. Whilst this result is not surprising given that the contagion mechanism is operating through the correlation channel the joint tests nonetheless still exhibit power for this experiment. From a comparison of the joint and single channel contagion tests, it would suggest that it is the interaction terms in the joint tests which is enabling the joint tests to identify indirectly the contagion channel.

3.2.4 Experiment IV

The final experiment combines experiments I and III by allowing for multiple contagion channels through the cokurtosis parameter θ_8 and the correlation parameter ρ. The joint density function is

$$f(r_i, r_j) = \exp\left[-\left(\frac{r_i^2 + r_j^2 - 2\rho r_i r_j}{2(1-\rho^2)}\right) - \theta_8 r_i^2 r_j^2 - \eta\right], \tag{25}$$

with respective values

$$\theta_8 = \{0.0, -0.1, -0.2, -0.3, -0.4, -0.5, -0.6, -0.7, -0.8, -0.9\},$$

and

$$\rho = \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}.$$

The noncrisis period corresponds to the parameter pair of $\theta_8 = 0.0$ and $\rho = 0.0$. For the crisis period the θ_8 and ρ parameters increase in magnitude with the first pair of parameters being $\theta_8 = -0.1$ and $\rho = 0.1$.

The results of the multiple contagion experiment are given in the fourth block of Table 2. The power function of the joint contagion test $JOINT$ exhibits monotonically increasing power reaching a power of 1.0 for the parameter pair $\theta_8 = -0.6$ and $\rho = 0.6$. This is supported by the CV_{22} and FR tests which exhibit good power properties as well. In fact, these single contagion channel tests yield marginally higher power than the joint test $JOINT$ which as in previous experiments is a reflection of the properties of the joint test testing across active and inactive contagion channels.

An interesting result in Table 2 is that the cokurtosis single contagion channel statistic based on CK_{31} also has a monotonically increasing power function, albeit at a slower rate than the $JOINT$ test and the two single channel tests CV_{22} and FR. This result is especially interesting given the fact that in the single channel experiments where the contagious channel operated through θ_8 in Experiment I and through ρ in Experiment III, the CK_{31} tests exhibit no power. This suggests that for this multiple channel contagion experiment the combination of increasing covolatility and correlation, the CK_{31} statistic is nonetheless able to identify increasing levels of contagion even though the true contagion channel is not arising from the interaction of skewness from one asset market and the mean from another asset market.

In contrast to the power properties of these even-ordered moment tests, the odd-ordered moment tests based on the joint channel test $COSKEW$ has either no or very little power, which again is to be expected given that the active contagion channels are based on even and not odd ordered moments. A similar result also occurs for the single channel coskewness test CS_{21}.

4 Application to Eurozone Equity Markets

The joint contagion tests in equations (14) and (16), as well as the single channel tests in equations (17) to (20) are now applied to identifying contagion in equity markets in the Eurozone during three periods of financial crises: the subprime crisis, the GFC and the European debt crisis. Ten representative countries from within the Eurozone are chosen for the empirical analysis and are Austria, Belgium, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain.[2] Two types of contagion are investigated. The first represents global contagion where the external effects of shocks in global equity markets, taken to be the US equity market, on the Eurozone equity markets are identified. The second is regional contagion where it is the effects of shocks internal to the Eurozone that are identified. For this application Germany is chosen to be the conduit market of contagion. The choice of the US and German equity markets as the source of contagion is consistent with the centre and periphery arguments that a crisis transmits across asset markets through a major financial centre, even if that financial centre does not appear to be affected by the crisis (Kaminsky and Reinhart (2003)).

The data are daily percentage equity returns on the 10 countries in the Eurozone plus the US. The data are plotted in Figure 1 which begins January 4, 2005 and ends November 28, 2014.[3] The shaded areas on the figure correspond to three financial market crisis periods beginning with the subprime crisis, followed by the GFC and the European debt crisis respectively, while the non shaded area is the non crisis period. The shading demarcates the noncrisis and crisis dates which are: January 4, 2005 to July 25, 2007 for the noncrisis period; July 26, 2007 to September 14, 2008 for the subprime crisis; September 15, 2008 to December 31, 2009 for the GFC; and from January 1, 2010 to November 28, 2014 for the European debt crisis.[4]

Summary statistics of the country equity returns are provided in Table 3 for the noncrisis period (January 4, 2005 to July 25, 2007) and the total crisis period (July 26,

[2]Not all Eurozone countries are included in the analysis. Seven countries were omitted as they joined the Eurozone after 2005 where our starts. These are Cyprus (2008), Estonia (2011), Latvia (2014), Lithuania (2015), Malta (2008), Slovakia (2009) and Slovenia (2007). Finland and Luxembourg were also excluded as they were not considered to be key crisis countries.

[3]The data source is Datastream. The mnemonics are: Austria - Austrian Traded index (ATXINDEX); Belgium - BEL 20 price index (BGBEL20); France - France CAC 40 price index (FRCAC40); Germany - MDAX Frankfurt price index (MDAXIDX); Greece - Athex composite price index (GRAGENL); Italy - FTSE MIB price index (FTSEMIB); Ireland - Ireland Se Overall price index (ISEQUIT); The Netherlands - AEX price index (AMSTEOE); Portugal - Portual PSI All Share price index (POPSIGN); Spain - IBEX 35 price index (IBEX351); US - Dow Jones Industrials price index (DJINDUS).

[4]This choice of crisis periods is based on a combination of institutional features arising from particular crisis trigger events, endogenous structural break testing using a generalization of the Diebold and Chen (1996) test to a multivariate VAR setting. A likelihood ratio test is used with standard errors based on a Wild paired bootstrap to correct for heteroskedastic shocks and to preserve the contemporaneous correlation structure in equity returns across asset markets.

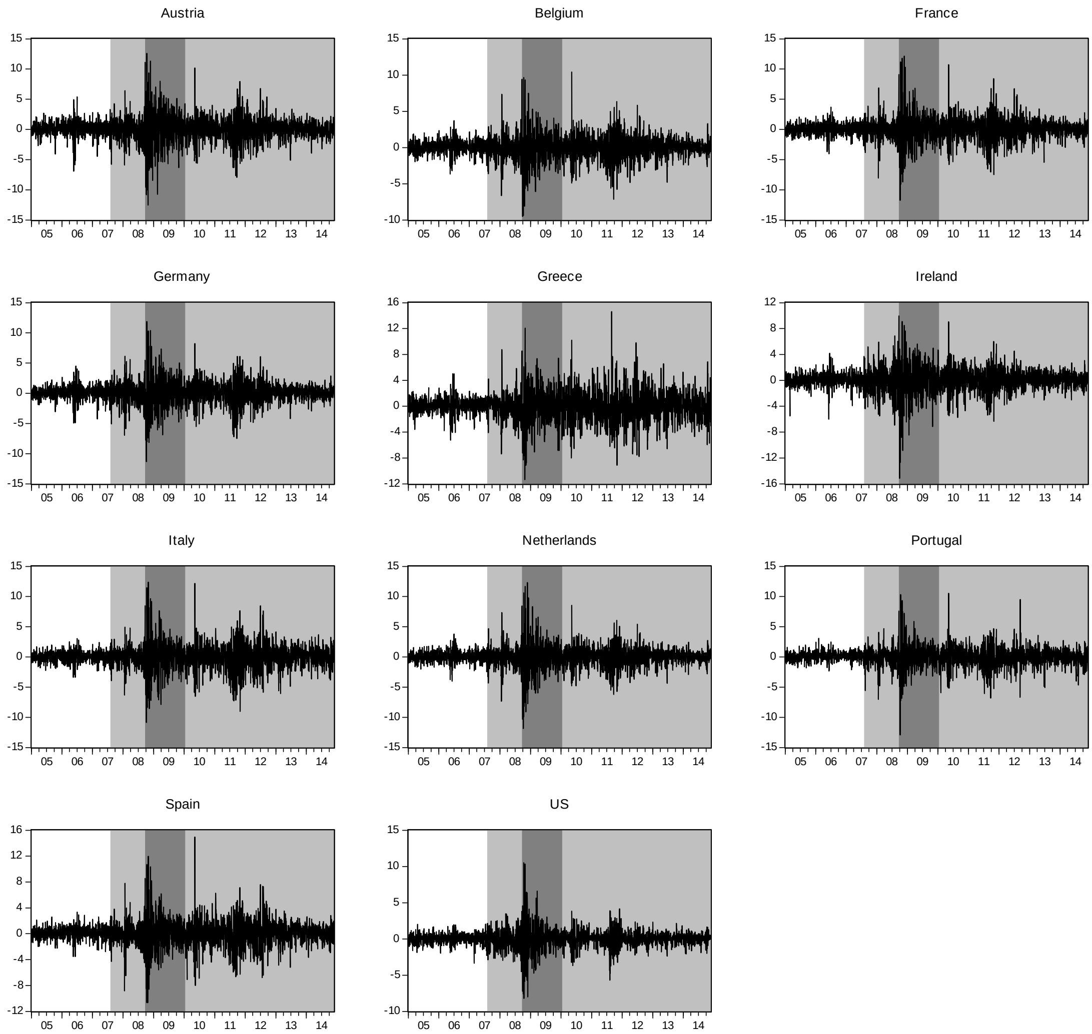

Figure 1: European and US percentage equity returns, January 4, 2005 to November 28, 2014. The figure highlights 4 distinct periods: the noncrisis period (January 12, 2005 to July 25, 2007), the subprime crisis (July 26, 2007 to September 14, 2008), the GFC (September 15, 2008 to December 31, 2009) and the European debt crisis (January 1, 2010 to November 28, 2014).

2007 to November 28, 2014) which encompasses the three crisis periods highlighted in Figure 1.[5] The crisis period is characterized by falls in the sample means and increases in the standard deviations when compared to the noncrisis period. There is a change in the returns distributions from negative skewness in the noncrisis period to either smaller negative skewness or even positive skewness in the crisis period. Except for Austria and Greece all countries experience increases in kurtosis which is also reflected by all countries experiencing larger (absolute) returns.

Table 4 provides statistics on the comoments between equity returns in the US and the 10 European equity markets during the noncrisis and crisis periods. The crisis period is characterized by higher levels of correlations than in the noncrisis period. The statistics for coskewness change depend upon which asset is defined as the squared term in the coskewness calculation. Defining coskewness between equity returns in the US and squared equity returns in Europe (r_i^1, r_j^2), which is the first coskewness term in the table, all coskewness coefficients are less negative in the crisis period compared to the noncrisis period with the exceptions of Germany, Ireland and the Netherlands. Reversing this relationship so that coskewness is between squared US equity returns and the Eurozone returns (r_i^2, r_j^1), almost all of the coefficients become more negative. The only exception is Portugal where coskewness becomes only marginally less negative in the crisis period. All European countries experience large increases in both types of cokurtosis as well as covolatility with the US during the crisis period.

4.1 Global Contagion

The empirical results of testing for global contagion in equity markets from the US to the Eurozone countries are presented in Table 5 for the three financial crises. To save space only the p-values of each test are reported.

The results for the subprime crisis given in the first block of Table 5 show little or no evidence of contagion through higher order moments for most countries. The exceptions are Italy where $JOINT$ is statistically significant at the 5% level with a p-value of 0.026, and to a lesser extent Ireland where it is only statistically significant at the 10% level. In the case of Italy as the $COSKEW$ statistic is statistically insignificant this suggests that the higher order moment effects are coming from the fourth order

[5] Time zones are accounted for by adjusting the returns using a two day rolling average of each return. Market fundamentals are accounted for by using the residuals of a VAR(5) estimated for all markets over the sample period (Forbes and Rigobon (2002), Fry, Martin and Tang (2010) and Fry-McKibbin and Hsiao (2016)).

Table 3:

Summary statistics of European and US percentage equity returns in the noncrisis and crisis periods. The returns are adjusted for time zone differences and autocorrelation.

Country	Mean	Min	Max	Std dev	Skewness	Kurtosis
Noncrisis: January 12, 2005 to July 25, 2007						
Austria	0.103	-6.923	5.383	1.166	-0.730	7.891
Belgium	0.065	-3.643	3.716	0.877	-0.234	4.570
France	0.065	-4.037	3.698	0.925	-0.234	4.198
Germany	0.107	-4.931	4.492	1.057	-0.565	6.231
Greece	0.090	-5.262	5.015	1.100	-0.372	5.820
Ireland	0.054	-6.016	4.193	0.995	-0.654	7.159
Italy	0.043	-3.364	3.069	0.874	-0.280	3.900
Netherlands	0.069	-4.000	3.804	0.864	-0.173	4.765
Portugal	0.090	-2.340	3.086	0.699	0.078	4.063
Spain	0.077	-3.537	3.367	0.894	-0.187	4.164
US	0.038	-3.349	2.069	0.633	-0.333	4.718
Crisis: July 26, 2007 to November 28, 2014						
Austria	-0.044	-12.536	12.611	2.129	-0.147	7.652
Belgium	-0.021	-9.507	10.440	1.736	-0.117	7.414
France	-0.020	-11.737	12.143	1.930	0.074	8.496
Germany	0.018	-11.326	11.887	1.913	-0.113	7.135
Greece	-0.092	-11.366	14.637	2.382	0.074	5.480
Ireland	-0.033	-15.151	9.950	1.940	-0.537	8.422
Italy	-0.042	-10.864	12.381	2.115	-0.026	6.749
Netherlands	-0.018	-11.856	12.316	1.817	-0.073	10.155
Portugal	-0.052	-12.916	10.530	1.689	-0.104	8.713
Spain	-0.022	-10.657	14.968	2.036	0.098	7.925
US	0.013	-8.201	10.508	1.294	-0.058	12.593

Table 4:

Comoment statistics between the percentage equity returns of the US (i) and the Eurozone countries (j) during the noncrisis and crisis periods. All returns are adjusted for time zone differences and autocorrelation.

Country	Correlation	Coskewness		Cokurtosis		Covolatility
	(r_i^1, r_j^1)	(r_i^1, r_j^2)	(r_i^2, r_j^1)	(r_i^3, r_j^1)	(r_i^1, r_j^3)	(r_i^2, r_j^2)
		Noncrisis: January 12, 2005 to July 25, 2007				
Austria	0.252	-0.224	-0.222	1.720	1.240	1.550
Belgium	0.342	-0.133	-0.168	2.004	1.398	1.835
France	0.413	-0.086	-0.140	2.186	1.460	1.909
Germany	0.347	-0.112	-0.204	2.300	1.936	2.226
Greece	0.179	-0.124	-0.178	1.527	1.120	1.643
Ireland	0.256	-0.112	-0.152	1.649	1.707	1.728
Italy	0.385	-0.129	-0.174	2.076	1.342	1.771
Netherlands	0.394	-0.052	-0.107	1.948	1.449	1.745
Portugal	0.175	-0.104	-0.145	1.160	0.575	1.222
Spain	0.396	-0.111	-0.162	2.143	1.428	1.762
		Crisis: July 26, 2007 to November 28, 2014				
Austria	0.484	-0.101	-0.278	5.703	4.107	5.016
Belgium	0.557	-0.118	-0.294	6.140	4.330	5.160
France	0.577	-0.022	-0.207	5.896	4.781	5.131
Germany	0.558	-0.125	-0.248	5.956	4.590	5.130
Greece	0.307	-0.068	-0.202	3.206	1.956	2.400
Ireland	0.486	-0.392	-0.446	4.083	4.639	4.201
Italy	0.530	-0.002	-0.211	4.849	3.812	4.441
Netherlands	0.582	-0.166	-0.321	6.419	5.833	5.994
Portugal	0.429	-0.024	-0.139	4.593	3.911	4.186
Spain	0.528	-0.046	-0.187	5.048	4.191	4.327

Note: The comoment statistics between r_{it}^m and r_{jt}^n are computed as $T^{-1}\sum_{t=1}^{T} z_{i,t}^m z_{j,t}^n$, where $z_{i,t} = \left(r_{it} - \widehat{\mu}_i\right)/\widehat{\sigma}_i$ and $z_{j,t} = \left(r_{jt} - \widehat{\mu}_j\right)/\widehat{\sigma}_j$ are respectively the standardized returns for the US and the European markets.

moments which is confirmed by the single channel tests CV_{22} and CK_{13} which are both statistically significant. In contrast, the significance of $COSKEW$ for Ireland shows that contagion occurs through coskewness, which from the single channel coskewness statistic CS_{12}, arises from the effects of US equity returns on volatility in the Irish equity market. Whilst there is limited evidence of contagion in higher order moments during the subprime crisis, the results of the FR test suggest that contagion mainly operated through the correlation channel with Austria, France, Germany, Italy, the Netherlands and Spain showing a significant change in correlations at the 5% level, and weaker evidence for Belgium where the test is only significant at the 10% level. Of all of the Eurozone countries, Greece and Portugal interestingly appear to be the only countries not to be affected by contagion (at the 5% level) during the subprime crisis.

The contagion test results for the GFC period are given in the second block of Table 5. A comparison of the subprime results (first block) and the GFC results (second block) reveals a dramatic change in the number and types of channels of contagion operating from the US to Europe. During the subprime crisis it was the correlation channels that were the active channels, whereas during the GFC higher order moment channels have become active as well. In particular, both joint tests, $JOINT$ and $COSKEW$, provide strong evidence of contagion transmitting via the third and fourth order moment channels for nearly all Eurozone countries. The one exception is Italy where the p-value on the $COSKEW$ statistic suggests that it is not the coskewness channels of contagion that are operating during the GFC, but rather it is fourth order moment channels which from the single channel statistics are the result of volatility spillovers (significant CV_{22} statistic) from the US to Eurozone equity markets and cokurtosis (significant CK_{13} and CK_{31} statistics). The FR correlation test also shows evidence of contagion from the US to some of the Eurozone countries. Interestingly, this set of Eurozone countries does not include Greece, Ireland and Portugal who all eventually experienced large contractions in their equity markets and economies in general during the European debt crisis. The fact that the FR correlation based test does not detect any evidence of contagion for these countries highlights the importance of the higher order moment contagion tests which find strong evidence of contagion during the GFC from the US for these countries.

The results of the contagion tests for the European debt crisis are given in the third block of Table 5. The joint test $JOINT$ finds very strong evidence of contagion from the US to all European countries. Inspection of the $COSKEW$ joint test results suggests

Table 5:

Tests of contagion from the US to the Eurozone countries for selected crisis periods: p-values reported with values less than 0.05, showing evidence of contagion at the 5% level. All tests are computed relative to the noncrisis period of January 5, 2005 to July 25, 2007.

Statistic	Austr.	Belg.	Fr.	Germ.	Gr.	Irel.	Italy	Neth.	Port.	Spain
				Subprime crisis: July 26, 2007 to September 14, 2008						
$JOINT$	0.589	0.444	0.386	0.305	0.760	0.094	0.026	0.421	0.213	0.129
$COSKEW$	0.196	0.576	0.360	0.424	0.813	0.039	0.157	0.906	0.377	0.292
CV_{22}	0.484	0.133	0.145	0.072	0.255	0.310	0.044	0.138	0.287	0.030
CK_{31}	0.817	0.098	0.247	0.388	0.973	0.125	0.341	0.114	0.116	0.252
CK_{13}	0.625	0.236	0.203	0.089	0.794	0.296	0.005	0.987	0.089	0.158
CS_{21}	0.170	0.448	0.595	0.301	0.744	0.101	0.398	0.701	0.369	0.244
CS_{12}	0.113	0.312	0.174	0.257	0.533	0.012	0.061	0.978	0.203	0.156
FR	0.045	0.053	0.000	0.000	0.524	0.604	0.001	0.000	0.148	0.000
				GFC: September 15, 2008 to December 31, 2009						
$JOINT$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$COSKEW$	0.095	0.002	0.009	0.027	0.003	0.000	0.209	0.000	0.021	0.017
CV_{22}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CK_{31}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CK_{13}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CS_{21}	0.048	0.013	0.013	0.028	0.043	0.000	0.090	0.000	0.036	0.023
CS_{12}	0.104	0.001	0.006	0.017	0.001	0.000	0.242	0.000	0.012	0.010
FR	0.106	0.003	0.000	0.001	0.860	0.066	0.000	0.000	0.767	0.000
				European debt crisis: January 1, 2010 to November 28, 2014						
$JOINT$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$COSKEW$	0.313	0.306	0.558	0.867	0.884	0.249	0.122	0.954	0.552	0.322
CV_{22}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CK_{31}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CK_{13}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CS_{21}	0.214	0.355	0.638	0.621	0.753	0.401	0.333	0.811	0.899	0.285
CS_{12}	0.196	0.139	0.324	0.655	0.811	0.496	0.050	0.961	0.381	0.135
FR	0.000	0.000	0.000	0.000	0.081	0.000	0.002	0.000	0.000	0.040

that these higher order moment effects are solely the result of volatility spillovers or cokurtosis, which is confirmed by the single channel contagion statistics CV_{22}, CK_{13} and CK_{31}. The Forbes-Rigobon statistic FR also finds significant evidence of contagion from the US through the correlation channel for all countries with one exception at the 5% level. The exception interestingly is Greece with a p-value of 0.081. This weaker evidence of contagion through the correlation channel is in stark contrast to the higher order moment results through covolatility and cokurtosis which are all found to be statistically important for Greece. If only traditional correlation measures of contagion are the focus, contagion to Greece would remain undetected.

4.2 Regional Contagion

The results of testing for regional contagion from Germany to the other countries within the Eurozone are presented in Table 6 for the three financial crises. The joint test $JOINT$ shows strong evidence of higher order moment contagion operating from Germany to all of the other Eurozone countries, which is also present during all three crisis periods. Upon closer inspection of the p-values associated with the joint and single channel tests, it is the fourth order comoments that tend to be the more pervasive channels operating.

The Forbes-Rigobon FR statistic also detects strong evidence of regional contagion through the correlation channel for nearly all countries during the subprime crisis and the GFC, but surprisingly the effects are limited during the European debt crisis. During this latter period it is Greece, Italy and Spain (at the 5% level) that are affected by Germany through the correlation channel, with weaker links for France and the Netherlands (at the 10% level).

5 Conclusions

This paper provided a new class of joint tests of contagion that explicitly allowed for simultaneous contagious linkages connecting asset markets through higher order comoments. The tests were designed to have power where contagion operated through coskewness, cokurtosis and covolatility. The finite sample properties of the proposed tests were investigated using a range of Monte Carlo experiments and compared to existing tests of contagion which primarily focussed on single comoment channels.

The new tests were applied to identifying contagion in Eurozone equity markets

Table 6:

Tests of contagion from Germany to the Eurozone countries for selected crisis periods: p-values reported with values less than 0.05, showing evidence of contagion at the 5% level. All tests are computed relative to the noncrisis period of January 5, 2005 to July 25, 2007.

Statistic	Austr.	Belg.	Fr.	Gr.	Irel.	Italy	Neth.	Port.	Spain
Subprime crisis: July 26, 2007 to September 14, 2008									
$JOINT$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$COSKEW$	0.027	0.012	0.007	0.278	0.027	0.020	0.003	0.014	0.000
CV_{22}	0.000	0.027	0.924	0.000	0.000	0.048	0.086	0.000	0.098
CK_{31}	0.000	0.000	0.006	0.000	0.000	0.173	0.001	0.339	0.036
CK_{13}	0.000	0.747	0.050	0.001	0.004	0.001	0.543	0.000	0.000
CS_{21}	0.009	0.270	0.241	0.408	0.026	0.237	0.658	0.828	0.391
CS_{12}	0.031	0.736	0.936	0.829	0.333	0.857	0.298	0.115	0.240
FR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.000
GFC: September 15, 2008 to December 31, 2009									
$JOINT$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$COSKEW$	0.337	0.032	0.099	0.089	0.000	0.328	0.000	0.020	0.791
CV_{22}	0.000	0.075	0.234	0.000	0.000	0.016	0.005	0.000	0.334
CK_{31}	0.000	0.155	0.110	0.000	0.652	0.553	0.462	0.002	0.172
CK_{13}	0.000	0.000	0.014	0.453	0.000	0.001	0.000	0.000	0.016
CS_{21}	0.187	0.550	0.595	0.838	0.090	0.995	0.177	0.166	0.783
CS_{12}	0.198	0.076	0.178	0.103	0.000	0.469	0.003	0.011	0.941
FR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.064	0.000
European debt crisis: January 1, 2010 to November 28, 2014									
$JOINT$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$COSKEW$	0.001	0.037	0.088	0.125	0.087	0.033	0.028	0.497	0.078
CV_{22}	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CK_{31}	0.183	0.004	0.000	0.002	0.000	0.000	0.000	0.000	0.000
CK_{13}	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CS_{21}	0.000	0.124	0.071	0.182	0.046	0.039	0.211	0.576	0.033
CS_{12}	0.000	0.460	0.186	0.066	0.175	0.201	0.703	0.923	0.068
FR	0.640	0.979	0.087	0.000	0.469	0.000	0.081	0.962	0.000

during the three financial crises, beginning with the subprime crisis, the GFC and more recently the European debt crisis. Both global and regional contagious linkages were tested. Using daily data on equity returns from 2005 to 2014, the empirical results showed significant higher order comoment contagion operating throughout all three crisis periods. For some countries the traditional measure of contagion based on correlations failed to detect evidence of contagion during some of the financial crises when contagion operated through the higher order moment channels.

A Derivations of Test Statistics

This appendix provides the derivations of the joint tests of contagion. Both statistics are based on the following bivariate generalized normal distribution

$$\begin{aligned} f(r_{it}, r_{jt}) &= \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2 + \left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2\right.\right. \\ &\quad \left. - 2\rho\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)\right) \\ &\quad +\theta_4\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 + \theta_5\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1 \\ &\quad +\theta_6\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^3 + \theta_7\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^3\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1 \\ &\quad \left. +\theta_8\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 - \eta_t\right], \end{aligned} \tag{26}$$

where η is the normalizing constant

$$\begin{aligned} \eta_t &= \ln\iint \exp\left[-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2 + \left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2\right.\right. \\ &\quad \left. - 2\rho\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)\right) \\ &\quad +\theta_4\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 + \theta_5\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1 \\ &\quad +\theta_6\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^3 + \theta_7\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^3\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1 \\ &\quad \left. +\theta_8\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2\right] dr_{i,t}dr_{j,t}, \\ &= \ln\iint \exp\left(h_t\right) dr_{i,t}dr_{j,t}. \end{aligned} \tag{27}$$

Let the exponent of (26) be defined as

$$
\begin{aligned}
h_t &= -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2 + \left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 - 2\rho\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)\right) \qquad (28)\\
&+\theta_4\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 + \theta_5\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1\\
&+\theta_6\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^3 + \theta_7\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^3\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1\\
&+\theta_8\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2.
\end{aligned}
$$

A.1 Information Matrix

Under the null hypothesis of bivariate normality the following elements of the information matrix are used in the derivations of the joint tests of contagion

$$
\begin{aligned}
I_{1,1,t} &= E\left[\left(\frac{\partial h_t}{\partial \mu_i}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \mu_i}\right] = \frac{1}{\sigma_i^2}\left(\frac{1}{1-\rho^2}\right),\\
I_{1,2,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \mu_j}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \mu_j}\right] = -\frac{\rho}{\sigma_i\sigma_j}\left(\frac{1}{1-\rho^2}\right),\\
I_{1,3,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \sigma_i^2}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right] = 0,\\
I_{1,4,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \sigma_j^2}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] = 0,\\
I_{1,5,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \rho}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \rho}\right] = 0,\\
I_{1,6,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \theta_4}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \theta_4}\right] = \frac{1}{\sigma_i},\\
I_{1,7,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \theta_5}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \theta_5}\right] = \frac{2\rho}{\sigma_i},\\
I_{1,8,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \theta_6}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \theta_6}\right] = 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{1,9,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \theta_7}\right] = 0,\\
I_{1,10,t} &= E\left[\frac{\partial h_t}{\partial \mu_i}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \mu_i}\right]E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 0,\\
I_{2,2,t} &= E\left[\left(\frac{\partial h_t}{\partial \mu_j}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \mu_j}\right] = \frac{1}{\sigma_j^2}\left(\frac{1}{1-\rho^2}\right),\\
I_{2,3,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \sigma_i^2}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right] = 0,\\
I_{2,4,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \sigma_j^2}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] = 0,\\
I_{2,5,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \rho}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \rho}\right] = 0,\\
I_{2,6,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \theta_4}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \theta_4}\right] = \frac{2\rho}{\sigma_j},\\
I_{2,7,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \theta_5}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \theta_5}\right] = \frac{1}{\sigma_j},\\
I_{2,8,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \theta_6}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \theta_6}\right] = 0,\\
I_{2,9,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \theta_7}\right] = 0,\\
I_{2,10,t} &= E\left[\frac{\partial h_t}{\partial \mu_j}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \mu_j}\right]E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 0,\\
I_{3,3,t} &= E\left[\left(\frac{\partial h_t}{\partial \sigma_i^2}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right]E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right] = \frac{2-\rho^2}{4\sigma_i^4}\left(\frac{1}{1-\rho^2}\right),\\
I_{3,4,t} &= E\left[\frac{\partial h_t}{\partial \sigma_i^2}\frac{\partial h_t}{\partial \sigma_j^2}\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right]E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] = \frac{-\rho^2}{4\sigma_i^2\sigma_j^2}\left(\frac{1}{1-\rho^2}\right),\\
I_{3,5,t} &= E\left[\frac{\partial h_t}{\partial \sigma_i^2}\frac{\partial h_t}{\partial \rho}\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right]E\left[\frac{\partial h_t}{\partial \rho}\right] = \frac{-\rho}{2\sigma_i^2}\left(\frac{1}{1-\rho^2}\right),\\
I_{3,6,t} &= E\left[\frac{\partial h_t}{\partial \sigma_i^2}\frac{\partial h_t}{\partial \theta_4}\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right]E\left[\frac{\partial h_t}{\partial \theta_4}\right] = 0,\\
I_{3,7,t} &= E\left[\frac{\partial h_t}{\partial \sigma_i^2}\frac{\partial h_t}{\partial \theta_5}\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right]E\left[\frac{\partial h_t}{\partial \theta_5}\right] = 0,\\
I_{3,8,t} &= E\left[\frac{\partial h_t}{\partial \sigma_i^2}\frac{\partial h_t}{\partial \theta_6}\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right]E\left[\frac{\partial h_t}{\partial \theta_6}\right] = \frac{3\rho}{2\sigma_i^2},\\
I_{3,9,t} &= E\left[\frac{\partial h_t}{\partial \sigma_i^2}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right]E\left[\frac{\partial h_t}{\partial \theta_7}\right] = \frac{9\rho}{2\sigma_i^2},
\end{aligned}
$$

$$
\begin{aligned}
I_{3,10,t} &= E\left[\frac{\partial h_t}{\partial \sigma_i^2}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \sigma_i^2}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = \frac{2\rho^2+1}{\sigma_i^2}, \\
I_{4,4,t} &= E\left[\left(\frac{\partial h_t}{\partial \sigma_j^2}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] = \frac{2-\rho^2}{4\sigma_j^4}\left(\frac{1}{1-\rho^2}\right), \\
I_{4,5,t} &= E\left[\frac{\partial h_t}{\partial \sigma_j^2}\frac{\partial h_t}{\partial \rho}\right] - E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] E\left[\frac{\partial h_t}{\partial \rho}\right] = -\frac{\rho}{2\sigma_j^2}\left(\frac{1}{1-\rho^2}\right), \\
I_{4,6,t} &= E\left[\frac{\partial h_t}{\partial \sigma_j^2}\frac{\partial h_t}{\partial \theta_4}\right] - E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] E\left[\frac{\partial h_t}{\partial \theta_4}\right] = 0, \\
I_{4,7,t} &= E\left[\frac{\partial h_t}{\partial \sigma_j^2}\frac{\partial h_t}{\partial \theta_5}\right] - E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] E\left[\frac{\partial h_t}{\partial \theta_5}\right] = 0, \\
I_{4,8,t} &= E\left[\frac{\partial h_t}{\partial \sigma_j^2}\frac{\partial h_t}{\partial \theta_6}\right] - E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] E\left[\frac{\partial h_t}{\partial \theta_6}\right] = \frac{9\rho}{2\sigma_j^2}, \\
I_{4,9,t} &= E\left[\frac{\partial h_t}{\partial \sigma_j^2}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] E\left[\frac{\partial h_t}{\partial \theta_7}\right] = \frac{3\rho}{2\sigma_j^2}, \\
I_{4,10,t} &= E\left[\frac{\partial h_t}{\partial \sigma_j^2}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \sigma_j^2}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = \frac{2\rho^2+1}{\sigma_j^2}, \\
I_{5,5,t} &= E\left[\left(\frac{\partial h_t}{\partial \rho}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \rho}\right] E\left[\frac{\partial h_t}{\partial \rho}\right] = \frac{1+\rho^2}{1-\rho^2}\left(\frac{1}{1-\rho^2}\right), \\
I_{5,6,t} &= E\left[\frac{\partial h_t}{\partial \rho}\frac{\partial h_t}{\partial \theta_4}\right] - E\left[\frac{\partial h_t}{\partial \rho}\right] E\left[\frac{\partial h_t}{\partial \theta_4}\right] = 0, \\
I_{5,7,t} &= E\left[\frac{\partial h_t}{\partial \rho}\frac{\partial h_t}{\partial \theta_5}\right] - E\left[\frac{\partial h_t}{\partial \rho}\right] E\left[\frac{\partial h_t}{\partial \theta_5}\right] = 0, \\
I_{5,8,t} &= E\left[\frac{\partial h_t}{\partial \rho}\frac{\partial h_t}{\partial \theta_6}\right] - E\left[\frac{\partial h_t}{\partial \rho}\right] E\left[\frac{\partial h_t}{\partial \theta_6}\right] = 3, \\
I_{5,9,t} &= E\left[\frac{\partial h_t}{\partial \rho}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \rho}\right] E\left[\frac{\partial h_t}{\partial \theta_7}\right] = 3, \\
I_{5,10,t} &= E\left[\frac{\partial h_t}{\partial \rho}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \rho}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 4\rho, \\
I_{6,6,t} &= E\left[\left(\frac{\partial h_t}{\partial \theta_4}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \theta_4}\right] E\left[\frac{\partial h_t}{\partial \theta_4}\right] = 3+12\rho^2, \\
I_{6,7,t} &= E\left[\frac{\partial h_t}{\partial \theta_4}\frac{\partial h_t}{\partial \theta_5}\right] - E\left[\frac{\partial h_t}{\partial \theta_4}\right] E\left[\frac{\partial h_t}{\partial \theta_5}\right] = 6\rho^3+9\rho, \\
I_{6,8,t} &= E\left[\frac{\partial h_t}{\partial \theta_4}\frac{\partial h_t}{\partial \theta_6}\right] - E\left[\frac{\partial h_t}{\partial \theta_4}\right] E\left[\frac{\partial h_t}{\partial \theta_6}\right] = 0, \\
I_{6,9,t} &= E\left[\frac{\partial h_t}{\partial \theta_4}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \theta_4}\right] E\left[\frac{\partial h_t}{\partial \theta_7}\right] = 0, \\
I_{6,10,t} &= E\left[\frac{\partial h_t}{\partial \theta_4}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \theta_4}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 0,
\end{aligned}
$$

$$
\begin{aligned}
I_{7,7,t} &= E\left[\left(\frac{\partial h_t}{\partial \theta_5}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \theta_5}\right] E\left[\frac{\partial h_t}{\partial \theta_5}\right] = 3 + 12\rho^2, \\
I_{7,8,t} &= E\left[\frac{\partial h_t}{\partial \theta_5}\frac{\partial h_t}{\partial \theta_6}\right] - E\left[\frac{\partial h_t}{\partial \theta_5}\right] E\left[\frac{\partial h_t}{\partial \theta_6}\right] = 0, \\
I_{7,9,t} &= E\left[\frac{\partial h_t}{\partial \theta_5}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \theta_5}\right] E\left[\frac{\partial h_t}{\partial \theta_7}\right] = 0, \\
I_{7,10,t} &= E\left[\frac{\partial h_t}{\partial \theta_5}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \theta_5}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 0, \\
I_{8,8,t} &= E\left[\left(\frac{\partial h_t}{\partial \theta_6}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \theta_6}\right] E\left[\frac{\partial h_t}{\partial \theta_6}\right] = 15 + 81\rho^2, \\
I_{8,9,t} &= E\left[\frac{\partial h_t}{\partial \theta_6}\frac{\partial h_t}{\partial \theta_7}\right] - E\left[\frac{\partial h_t}{\partial \theta_6}\right] E\left[\frac{\partial h_t}{\partial \theta_7}\right] = 24\rho^4 + 63\rho^2 + 9, \\
I_{8,10,t} &= E\left[\frac{\partial h_t}{\partial \theta_6}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \theta_6}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 54\rho^3 + 42\rho, \\
I_{9,9,t} &= E\left[\left(\frac{\partial h_t}{\partial \theta_7}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \theta_7}\right] E\left[\frac{\partial h_t}{\partial \theta_7}\right] = 15 + 81\rho^2, \\
I_{9,10,t} &= E\left[\frac{\partial h_t}{\partial \theta_7}\frac{\partial h_t}{\partial \theta_8}\right] - E\left[\frac{\partial h_t}{\partial \theta_7}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 54\rho^3 + 42\rho, \\
I_{10,10,t} &= E\left[\left(\frac{\partial h_t}{\partial \theta_8}\right)^2\right] - E\left[\frac{\partial h_t}{\partial \theta_8}\right] E\left[\frac{\partial h_t}{\partial \theta_8}\right] = 8 + 68\rho^2 + 20\rho^4.
\end{aligned}
$$

A.2 Joint Contagion Test

From (26) and (27) the joint test of contagion ($JOINT$) through covolatility, cokurtosis and coskewness is based on the null hypothesis

$$
H_0 : \theta_4 = \theta_5 = \theta_6 = \theta_7 = \theta_8 = 0. \tag{29}
$$

Under the null hypothesis of bivariate normality, the maximum likelihood estimators of the unknown parameters are simply

$$
\widehat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} r_{it}, \qquad \widehat{\mu}_j = \frac{1}{T}\sum_{t=1}^{T} r_{jt}, \qquad \widehat{\sigma}_i^2 = \frac{1}{T}\sum_{t=1}^{T} \left(r_{it} - \widehat{\mu}_i\right)^2, \qquad \widehat{\sigma}_j^2 = \frac{1}{T}\sum_{t=1}^{T} \left(r_{jt} - \widehat{\mu}_j\right)^2, \tag{30}
$$

and for the the correlation parameter

$$
\widehat{\rho} = \frac{1}{T}\sum_{t=1}^{T} \left(\frac{r_{it} - \mu_i}{\sigma_i}\right)\left(\frac{r_{jt} - \mu_j}{\sigma_j}\right).
$$

Let the parameters of (26) be $\Theta = \{\mu_i, \mu_j, \sigma_i^2, \sigma_j^2, \rho, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8\}$. The information matrix under the null hypothesis ($H_0 : \theta_4 = \theta_5 = \theta_6 = \theta_7 = \theta_8 = 0$) is

$$
\begin{aligned}
I(\Theta) &= \left(E\left[\frac{\partial h}{\partial \Theta}\frac{\partial h}{\partial \Theta'}\right]\Bigg|_{\theta_j=0} - E\left[\frac{\partial h}{\partial \Theta}\right]\Bigg|_{\theta_j=0} E\left[\frac{\partial h}{\partial \Theta'}\right]\Bigg|_{\theta_j=0}\right), \forall j = 4, \ldots, 8, \\
&= \left(\frac{1}{1-\rho^2}\right)
\begin{bmatrix}
\frac{1}{\sigma_i^2} & \frac{-\rho}{\sigma_i\sigma_j} & 0 & 0 & 0 & \frac{(1-\rho^2)}{\sigma_i} & \frac{2\rho(1-\rho^2)}{\sigma_i} & 0 & 0 & 0 \\
\frac{-\rho}{\sigma_i\sigma_j} & \frac{1}{\sigma_j^2} & 0 & 0 & 0 & \frac{2\rho(1-\rho^2)}{\sigma_j} & \frac{(1-\rho^2)}{\sigma_j} & 0 & 0 & 0 \\
0 & 0 & \frac{2-\rho^2}{4\sigma_i^4} & \frac{-\rho^2}{4\sigma_i^2\sigma_j^2} & \frac{-\rho}{2\sigma_i^2} & 0 & 0 & \frac{3\rho(1-\rho^2)}{2\sigma_i^2} & \frac{9\rho(1-\rho^2)}{2\sigma_i^2} & \frac{(2\rho^2+1)(1-\rho^2)}{\sigma_i^2} \\
0 & 0 & \frac{-\rho^2}{4\sigma_i^2\sigma_j^2} & \frac{2-\rho^2}{4\sigma_j^4} & \frac{-\rho}{2\sigma_j^2} & 0 & 0 & \frac{9\rho(1-\rho^2)}{2\sigma_j^2} & \frac{3\rho(1-\rho^2)}{2\sigma_j^2} & \frac{(2\rho^2+1)(1-\rho^2)}{\sigma_j^2} \\
0 & 0 & \frac{-\rho}{2\sigma_i^2} & \frac{-\rho}{2\sigma_j^2} & \frac{1+\rho^2}{1-\rho^2} & 0 & 0 & 3(1-\rho^2) & 3(1-\rho^2) & 4\rho(1-\rho^2) \\
\frac{(1-\rho^2)}{\sigma_i} & \frac{2(1-\rho^2)\rho}{\sigma_j} & 0 & 0 & 0 & (3+12\rho^2)(1-\rho^2) & (9\rho+6\rho^3)(1-\rho^2) & 0 & 0 & 0 \\
\frac{2(1-\rho^2)\rho}{\sigma_i} & \frac{(1-\rho^2)}{\sigma_j} & 0 & 0 & 0 & (9\rho+6\rho^3)(1-\rho^2) & (3+12\rho^2)(1-\rho^2) & 0 & 0 & 0 \\
0 & 0 & \frac{3\rho(1-\rho^2)}{2\sigma_i^2} & \frac{9\rho(1-\rho^2)}{2\sigma_j^2} & 3(1-\rho^2) & 0 & 0 & (15+81\rho^2)(1-\rho^2) & (24\rho^4+63\rho^2+9)(1-\rho^2) & (54\rho^3+42\rho)(1-\rho^2) \\
0 & 0 & \frac{9\rho(1-\rho^2)}{2\sigma_i^2} & \frac{3\rho(1-\rho^2)}{2\sigma_j^2} & 3(1-\rho^2) & 0 & 0 & (24\rho^4+63\rho^2+9)(1-\rho^2) & (15+81\rho^2)(1-\rho^2) & (54\rho^3+42\rho)(1-\rho^2) \\
0 & 0 & \frac{(2\rho^2+1)(1-\rho^2)}{\sigma_i^2} & \frac{(2\rho^2+1)(1-\rho^2)}{\sigma_j^2} & 4\rho(1-\rho^2) & 0 & 0 & (54\rho^3+42\rho)(1-\rho^2) & (54\rho^3+42\rho)(1-\rho^2) & (8+68\rho^2+20\rho^4)(1-\rho^2)
\end{bmatrix}^{-1},
\end{aligned}
\tag{31}
$$

by using the elements of the information matrix given in Appendix A.1. Replacing the unknown population parameters by consistent estimators under the null hypothesis, the inverse asymptotic information matrix is

$$
I^{-1}(\widehat{\Theta}) = \left(1-\widehat{\rho}^2\right)
\begin{bmatrix}
\frac{1}{\widehat{\sigma}_i^2} & \frac{-\widehat{\rho}}{\widehat{\sigma}_i\widehat{\sigma}_j} & 0 & 0 & 0 & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i} & \frac{2\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i} & 0 & 0 & 0 \\
\frac{-\widehat{\rho}}{\widehat{\sigma}_i\widehat{\sigma}_j} & \frac{1}{\widehat{\sigma}_j^2} & 0 & 0 & 0 & \frac{2\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j} & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j} & 0 & 0 & 0 \\
0 & 0 & \frac{2-\widehat{\rho}^2}{4\widehat{\sigma}_i^4} & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_i^2\widehat{\sigma}_j^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_i^2} & 0 & 0 & \frac{3\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_i^2} & \frac{9\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_i^2} & \frac{\left(2\widehat{\rho}^2+1\right)\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i^2} \\
0 & 0 & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_i^2\widehat{\sigma}_j^2} & \frac{2-\widehat{\rho}^2}{4\widehat{\sigma}_j^4} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_j^2} & 0 & 0 & \frac{9\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_j^2} & \frac{3\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_j^2} & \frac{\left(2\widehat{\rho}^2+1\right)\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j^2} \\
0 & 0 & \frac{-\widehat{\rho}}{2\widehat{\sigma}_i^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_j^2} & \frac{1+\widehat{\rho}^2}{1-\widehat{\rho}^2} & 0 & 0 & 3\left(1-\widehat{\rho}^2\right) & 3\left(1-\widehat{\rho}^2\right) & 4\widehat{\rho}\left(1-\widehat{\rho}^2\right) \\
\frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i} & \frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_j} & 0 & 0 & 0 & \left(3+12\widehat{\rho}^2\right)\left(1-\widehat{\rho}^2\right) & \left(9\widehat{\rho}+6\widehat{\rho}^3\right)\left(1-\widehat{\rho}^2\right) & 0 & 0 & 0 \\
\frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_i} & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j} & 0 & 0 & 0 & \left(9\widehat{\rho}+6\widehat{\rho}^3\right)\left(1-\widehat{\rho}^2\right) & \left(3+12\widehat{\rho}^2\right)\left(1-\widehat{\rho}^2\right) & 0 & 0 & 0 \\
0 & 0 & \frac{3\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_i^2} & \frac{9\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_j^2} & 3\left(1-\widehat{\rho}^2\right) & 0 & 0 & \left(15+81\widehat{\rho}^2\right)\left(1-\widehat{\rho}^2\right) & \left(24\widehat{\rho}^4+63\widehat{\rho}^2+9\right)\left(1-\widehat{\rho}^2\right) & \left(54\widehat{\rho}^3+42\widehat{\rho}\right)\left(1-\widehat{\rho}^2\right) \\
0 & 0 & \frac{9\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_i^2} & \frac{3\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{2\widehat{\sigma}_j^2} & 3\left(1-\widehat{\rho}^2\right) & 0 & 0 & \left(24\widehat{\rho}^4+63\widehat{\rho}^2+9\right)\left(1-\widehat{\rho}^2\right) & \left(15+81\widehat{\rho}^2\right)\left(1-\widehat{\rho}^2\right) & \left(54\widehat{\rho}^3+42\widehat{\rho}\right)\left(1-\widehat{\rho}^2\right) \\
0 & 0 & \frac{\left(2\widehat{\rho}^2+1\right)\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i^2} & \frac{\left(2\widehat{\rho}^2+1\right)\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j^2} & 4\widehat{\rho}\left(1-\widehat{\rho}^2\right) & 0 & 0 & \left(54\widehat{\rho}^3+42\widehat{\rho}\right)\left(1-\widehat{\rho}^2\right) & \left(54\widehat{\rho}^3+42\widehat{\rho}\right)\left(1-\widehat{\rho}^2\right) & \left(8+68\widehat{\rho}^2+20\widehat{\rho}^4\right)\left(1-\widehat{\rho}^2\right)
\end{bmatrix}^{-1} . \qquad (32)
$$

Evaluating the gradients for θ_4, θ_5, θ_6, θ_7 and θ_8 under the null hypothesis gives

$$
\begin{aligned}
\frac{\partial \ln L(\theta)}{\partial \theta_4} &= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\partial h_t}{\partial \theta_4}\right) - \left(\frac{\partial \eta_t}{\partial \theta_4}\right) \\
&= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 - \left[E\left(\frac{\partial h_t}{\partial \theta_4}\right)\right] \\
&= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 - (0) \\
&= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2, \\
\frac{\partial \ln L(\theta)}{\partial \theta_5} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\partial h_t}{\partial \theta_5} - \frac{\partial \eta_t}{\partial \theta_5} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1, \\
\frac{\partial \ln L(\theta)}{\partial \theta_6} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\partial h_t}{\partial \theta_6} - \frac{\partial \eta_t}{\partial \theta_6} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^1\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^3 - 3\rho, \\
\frac{\partial \ln L(\theta)}{\partial \theta_7} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\partial h_t}{\partial \theta_7} - \frac{\partial \eta_t}{\partial \theta_7} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^3\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^1 - 3\rho, \\
\frac{\partial \ln L(\theta)}{\partial \theta_8} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\partial h_t}{\partial \theta_8} - \frac{\partial \eta_t}{\partial \theta_8} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\mu_i}{\sigma_i}\right)^2\left(\frac{r_{jt}-\mu_j}{\sigma_j}\right)^2 - \left(1+2\rho^2\right).
\end{aligned}
$$

The score function under H_0 is given by

$$
\begin{aligned}
G(\widehat{\Theta}) &= \left.\frac{\partial \ln L}{\partial \Theta}\right|_{\theta_j=0}, \qquad \forall j = 4, \ldots, 8, \\
&= \Big[\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
&\quad \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^2 \quad \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1 \\
&\quad \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^3 - 3\widehat{\rho} \\
&\quad \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^3\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1 - 3\widehat{\rho} \\
&\quad \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^2 - \left(1+2\widehat{\rho}^2\right)\ \Big]'.
\end{aligned}
\tag{33}
$$

The general form of the Lagrange multiplier statistic is given by

$$LM = TG(\widehat{\Theta})'I(\widehat{\Theta})^{-1}G(\widehat{\Theta}), \tag{34}$$

Substituting (33) and (32) into (34) gives the following LM statistic

$$
\begin{aligned}
LM = & \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^2}{\sqrt{\frac{2\left(1-\widehat{\rho}^6\right)}{T\left(2\widehat{\rho}^2+1\right)}}}\right)^2 + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1}{\sqrt{\frac{2\left(1-\widehat{\rho}^6\right)}{T\left(2\widehat{\rho}^2+1\right)}}}\right)^2 \\
& - \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^2}{\sqrt{\frac{1-\widehat{\rho}^6}{T\left(\widehat{\rho}^3+2\widehat{\rho}\right)}}}\right)\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1}{\sqrt{\frac{1-\widehat{\rho}^6}{T\left(\widehat{\rho}^3+2\widehat{\rho}\right)}}}\right) \\
& + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^3-3\widehat{\rho}}{\sqrt{\frac{6\left(\widehat{\rho}^{10}-\widehat{\rho}^8-\widehat{\rho}^2+1\right)}{T\left(3\widehat{\rho}^4+2\widehat{\rho}^2+1\right)}}}\right)^2 + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^3\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1-3\widehat{\rho}}{\sqrt{\frac{6\left(\widehat{\rho}^{10}-\widehat{\rho}^8-\widehat{\rho}^2+1\right)}{T\left(3\widehat{\rho}^4+2\widehat{\rho}^2+1\right)}}}\right)^2 \\
& + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^2-\left(1+2\widehat{\rho}^2\right)}{\sqrt{\frac{4\left(\widehat{\rho}^2-1\right)^2\left(\widehat{\rho}^4+1\right)}{T\left(\widehat{\rho}^4+6\widehat{\rho}^2+1\right)}}}\right)^2 \\
& + \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^3-3\widehat{\rho}}{\sqrt{\frac{3\left(\widehat{\rho}^{10}-\widehat{\rho}^8-\widehat{\rho}^2+1\right)}{T\left(\widehat{\rho}^6+2\widehat{\rho}^4+3\widehat{\rho}^2\right)}}}\right)\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^3\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1-3\widehat{\rho}}{\sqrt{\frac{3\left(\widehat{\rho}^{10}-\widehat{\rho}^8-\widehat{\rho}^2+1\right)}{T\left(\widehat{\rho}^6+2\widehat{\rho}^4+3\widehat{\rho}^2\right)}}}\right) \\
& - \left(\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^3-3\widehat{\rho}}{\sqrt{\frac{\left(\widehat{\rho}^2-1\right)^2\left(\widehat{\rho}^4+1\right)}{T\left(\widehat{\rho}^3+\widehat{\rho}\right)}}}\right)+\left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^3\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1-3\widehat{\rho}}{\sqrt{\frac{\left(\widehat{\rho}^2-1\right)^2\left(\widehat{\rho}^4+1\right)}{T\left(\widehat{\rho}^3+\widehat{\rho}\right)}}}\right)\right) \\
& \times \left(\frac{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^2-\left(1+2\widehat{\rho}^2\right)}{\sqrt{\frac{\left(\widehat{\rho}^2-1\right)^2\left(\widehat{\rho}^4+1\right)}{T\left(\widehat{\rho}^3+\widehat{\rho}\right)}}}\right).
\end{aligned}
\tag{35}
$$

The test statistic in (35) provides a general test of higher order moments for a particular sample period T. In the case of testing for contagion the approach is to follow Fry, Martin and Tang (2010) and construct a test statistic where the individual terms in (35) are replaced by the difference between each term evaluated using crisis data (y) and noncrisis data (x). As these expressions are based on estimating the correlation parameter ρ, this parameter is evaluated at the Forbes-Rigobon adjusted correlation statistic defined in (15) when the expression is evaluated using crisis data and by the unadjusted correlation coefficient when using noncrisis data. Constructing the joint statistic this way produces the statistic given in equation (14).

A.3 Joint Coskewness Test

A joint test of coskewness contagion is based on the null hypothesis

$$H_0 : \theta_4 = \theta_5 = 0, \tag{36}$$

in (26) and (27). Using the results of Appendix A.1 and replacing the unknown population parameters by consistent estimators under the null hypothesis, the inverse asymptotic information matrix under the null hypothesis ($H_0 : \theta_4 = \theta_5 = 0$) is

$$I^{-1}(\widehat{\Theta}) = \left(1-\widehat{\rho}^2\right)\begin{bmatrix} \frac{1}{\widehat{\sigma}_i^2} & \frac{-\widehat{\rho}}{\widehat{\sigma}_i\widehat{\sigma}_j} & 0 & 0 & 0 & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i} & \frac{2\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i} \\ \frac{-\widehat{\rho}}{\widehat{\sigma}_i\widehat{\sigma}_j} & \frac{1}{\widehat{\sigma}_j^2} & 0 & 0 & 0 & \frac{2\widehat{\rho}\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j} & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j} \\ 0 & 0 & \frac{2-\widehat{\rho}^2}{4\widehat{\sigma}_i^4} & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_i^2\widehat{\sigma}_j^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_i^2} & 0 & 0 \\ 0 & 0 & \frac{-\widehat{\rho}^2}{4\widehat{\sigma}_i^2\widehat{\sigma}_j^2} & \frac{2-\widehat{\rho}^2}{4\widehat{\sigma}_j^4} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_j^2} & 0 & 0 \\ 0 & 0 & \frac{-\widehat{\rho}}{2\widehat{\sigma}_i^2} & \frac{-\widehat{\rho}}{2\widehat{\sigma}_j^2} & \frac{1+\widehat{\rho}^2}{1-\widehat{\rho}^2} & 0 & 0 \\ \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_i} & \frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_j} & 0 & 0 & 0 & \left(3+12\widehat{\rho}^2\right)\left(1-\widehat{\rho}^2\right) & \left(9\widehat{\rho}+6\widehat{\rho}^3\right)\left(1-\widehat{\rho}^2\right) \\ \frac{2\left(1-\widehat{\rho}^2\right)\widehat{\rho}}{\widehat{\sigma}_i} & \frac{\left(1-\widehat{\rho}^2\right)}{\widehat{\sigma}_j} & 0 & 0 & 0 & \left(9\widehat{\rho}+6\widehat{\rho}^3\right)\left(1-\widehat{\rho}^2\right) & \left(3+12\widehat{\rho}^2\right)\left(1-\widehat{\rho}^2\right) \end{bmatrix}^{-1}. \tag{37}$$

The score function under H_0 is given as

$$\begin{aligned} G(\widehat{\Theta}) &= \left.\frac{\partial \ln L}{\partial \Theta}\right|_{\theta_4=\theta_5=0}, \\ &= \left[\begin{array}{c} 0 \quad 0 \quad 0 \quad 0 \quad 0 \\ \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^1\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^2 \\ \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{it}-\widehat{\mu}_i}{\widehat{\sigma}_i}\right)^2\left(\frac{r_{jt}-\widehat{\mu}_j}{\widehat{\sigma}_j}\right)^1 \end{array}\right]'. \end{aligned} \tag{38}$$

Substituting (37) and (38) into the Lagrange multiplier test in (34) gives

$$
\begin{aligned}
LM &= \left(\frac{\frac{1}{T} \sum_{t=1}^{T} \left(\frac{r_{it} - \widehat{\mu}_i}{\widehat{\sigma}_i} \right)^1 \left(\frac{r_{jt} - \widehat{\mu}_j}{\widehat{\sigma}_j} \right)^2}{\sqrt{\frac{\left(2 - 2\widehat{\rho}^6\right)}{T\left(2\widehat{\rho}^2 + 1\right)}}} \right)^2 + \left(\frac{\frac{1}{T} \sum_{t=1}^{T} \left(\frac{r_{it} - \widehat{\mu}_i}{\widehat{\sigma}_i} \right)^2 \left(\frac{r_{jt} - \widehat{\mu}_j}{\widehat{\sigma}_j} \right)^1}{\sqrt{\frac{\left(2 - 2\widehat{\rho}^6\right)}{T\left(2\widehat{\rho}^2 + 1\right)}}} \right)^2 \\
&\quad - \left(\frac{\frac{1}{T} \sum_{t=1}^{T} \left(\frac{r_{it} - \widehat{\mu}_i}{\widehat{\sigma}_i} \right)^1 \left(\frac{r_{jt} - \widehat{\mu}_j}{\widehat{\sigma}_j} \right)^2}{\sqrt{\frac{\left(1 - \widehat{\rho}^6\right)}{T\left(\widehat{\rho}^3 + 2\widehat{\rho}\right)}}} \right) \left(\frac{\frac{1}{T} \sum_{t=1}^{T} \left(\frac{r_{it} - \widehat{\mu}_i}{\widehat{\sigma}_i} \right)^2 \left(\frac{r_{jt} - \widehat{\mu}_j}{\widehat{\sigma}_j} \right)^1}{\sqrt{\frac{\left(1 - \widehat{\rho}^6\right)}{T\left(\widehat{\rho}^3 + 2\widehat{\rho}\right)}}} \right). \qquad (39)
\end{aligned}
$$

The test statistic in (39) provides a general test of coskewness for a particular sample period T. Following the approach adopted in Section A.2 a joint test of contagion based on coskewness is constructed by replacing each term in (39) by the difference between the term evaluated using crisis data (y) and noncrisis data (x). Again the correlation parameter is evaluated using the adjusted statistic in (15) for the crisis data and the standard unadjusted correlation coefficient using noncrisis data, which, in turn, yields the joint coskewness tests of contagion given in equation (16).

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