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Measuring Financial Interdependence in Asset Returns with an Application to Euro Zone Equities

Renée Fry-McKibbin
Crawford School of Public Policy, ANU and
Centre for Applied Macroeconomic Analysis, ANU

Cody Yu-Ling Hsiao
School of Business, Macau University of Science and Technology and
Centre for Applied Macroeconomic Analysis, ANU

Vance L. Martin
Department of Economics, The University of Melbourne and
Centre for Applied Macroeconomic Analysis, ANU

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Entropy theory, generalized exponential family, higher order comoment decomposition, independence testing

JEL Classification

C12, F30

Address for correspondence:

(E) cama.admin@anu.edu.au

Measuring Financial Interdependence in Asset Returns With an Application to Euro Zone Equities[*]

Renée Fry-McKibbin[%,#], Cody Yu-Ling Hsiao[ˆ,%] and Vance L. Martin[§]
[%]Centre for Applied Macroeconomic Analysis (CAMA)
[#]Crawford School of Public Policy, Australian National University
[ˆ]School of Business, Macau University of Science and Technology
[§]Department of Economics, The University of Melbourne

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Abstract

A general procedure is proposed to identify changes in asset return interdependence over time using entropy theory. The approach provides a decomposition of interdependence in terms of comoments including coskewness, cokurtosis and covolatility as well as more traditional measures based on second order moments such as correlations. A new diagnostic test of independence is also developed which incorporates these higher order comoments. The properties of the entropy interdependence measure are demonstrated using a number of simulation experiments, as well as applying the methodology to euro zone equity markets over the period 1990 to 2017.

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1 Introduction

Understanding the interdependence of asset returns is important in the construction of optimal well diversified portfolios (Markowitz (1959), Grubel (1968), Levy and Sarnat (1970)), risk management using multiple asset VaR measures (Jorion (1997), Basak and Shapiro (2001)) and pricing exchange options (Margrabe (1978)). Earlier approaches for measuring interdependence are based on bivariate correlations (Grubel (1968), Levy and Sarnat (1970), Grubel and Fadner (1971), King and Wadhwhani (1990)), spectral methods (Granger and Morgenstern (1970), Hilliard (1979)), then extended to multivariate dynamic models including vector autoregressions (Eun and Shim (1989)), vector error correction models (Arshanapalli and Doukas (1993)) and multivariate volatility (Wahab (2012), MacDonald, Sogiakas and Tsopanakis (2017)).

Latent factor models are also widely used for decomposing the determinants of asset return interdependence in terms of common and idiosyncratic factors beginning with Lessard (1974) and Solnik (1974), and more recently by Bekaert, Hodrick and Zhang (2009). Extensions to dynamic factors are by Haldane and Hall (1991) where the decomposition is in terms of regional and global factors and how this decomposition can change over time, and the connectedness models of Diebold and Yilmaz (2009,2014). In models of financial integration the decomposition ranges the spectrum from segmented markets where local and global asset markets operate independently, to the extreme case where asset markets are fully integrated (see Bekaert and Harvey (1995) and the applications of this model by Bekaert and Harvey (1997), Baele (2005), Hardouvelis, Malliaropulos and Priestley (2006,2007), Wang and Shih (2013), Abad, Chuliá and Gómez-Puig (2014), de Nicolò and Juvenal (2014) and Šimović, Tkalec, Vizek and Lee (2016)). Further extensions to modelling the joint movements of asset returns during financial crises and the identification of contagion include Bekaert and Harvey (2003), Dungey, Fry, González-Hermosillo and Martin (2005,2010), and Bekaert, Ehrmann, Fratzscher and Mehl (2014). In particular, Forbes and Rigobon (2002) argue that increases in asset return comovements during financial crises is due to changes in interdependence and not contagion.

A common theme in most of this previous work is the focus on measuring asset return comovements through the first and second order moments. However, higher order comoments including coskewness, cokurtosis and covolatility are also important in

modeling asset returns in general (Harvey and Siddique (2000), Christoffersen, Errunza, Jacobs and Langlois (2012), Kostakis, Muhammadb and Siganos (2012)), options (Fry-McKibbin, Martin and Tang (2014)), and contagion (Fry, Martin and Tang (2010), Fry-McKibbin and Hsiao (2017)). These higher order comoments allow for interactions between expected returns and volatility, between expected returns and skewness, as well as between the cross-volatilities of different asset markets. Tail correlations using copulas provide another approach to gauge the strength of joint extreme events (Huang, Liu, Rhee and Wub (2012)).

To capture the role of higher order comoments in the joint determination of asset markets the approach consists of specifying a flexible class of multivariate distributions based on the generalized exponential family (Lye and Martin (1993) and Fry, Martin and Tang (2010)) to construct a measure of interdependence using entropy theory. The proposed modelling framework has the dual advantage of not requiring the specification of a particular parametric model, or the need to choose a particular metric to measure the strength of alternative channels linking asset returns. A further advantage is that a new diagnostic test of independence is proposed which explicitly takes into account the role of higher order comoments. Maasoumi and Racine (2002) and Granger, Maasoumi and Racine (2004) also use entropy theory to measure interdependence, but choose a different metric while adopting a fully nonparametric specification of the underlying distribution. Their test statistic is computed using a nonparametric Gaussian kernel (see also Racine and Maasoumi (2007), Maasoumi and Racine (2008), Giannerine, Maasoumi and Dagum (2015)). The advantage of the present framework over these previous approaches is that the specification of the multivariate generalized normal distribution makes it possible to decompose asset comovements in terms of their various comoments. The generalized exponential family of distributions is also shown to represent a natural theoretical choice as it is shown to have maximum entropy given a set of higher order comoments.

To demonstrate the flexibility of the proposed measure of interdependence two simulation experiments are performed based on the Bekaert-Harvey (1995) model of financial integration and the Solnik-Roulet (2000) multiple asset model of cross-sectional interdependence. For both models the entropy interdependence measure successfully tracks the comovements in asset returns over time regardless of the type of data gen-

erating mechanism specified. The entropy measure is also applied to studying equity market interdependence of euro zone countries both globally and within Europe, from 1990 to 2017. The empirical results provide evidence that European equity markets operated independently of global asset markets prior to the adoption of the euro, but progressively became more interdependent over time with the level of interdependence peaking during the global financial crisis. The empirical results also show a fall in interdependence during the post-GFC period largely as a result of Greece, Ireland, Portugal and Spain becoming independent of global equity markets.

The rest of the paper proceeds as follows. Section 2 compares the statistical properties of equity return comovements in the euro zone which are used to motivate the entropy interdependence measure in Section 3. The properties of this measure are investigated in Section 4 using a range of simulation experiments, which is then applied in Section 5 to euro zone equity markets. Concluding comments are given in Section 6 with derivations and additional simulation results presented in the Appendices.

2   Equity Comovements

To motivate the use of the entropy measure of interdependence, Figures 1 and 2 provide scatter plots of the log-returns of equities between euro zone countries and the U.S.. The euro zone countries consist of Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. With the exception of Greece, these countries are the original adopters of the euro on January 1, 1999, with Greece adopting the euro shortly after in 2001.[1]

Equity returns are computed as the change in the log-prices with all series denominated in US dollars. All equity returns are computed on a weekly basis to circumvent non-synchronous trading issues across asset markets and expressed as a percentage. The sample period ranges from January 12, 1990 and ends May 5, 2017, a total of $T = 1427$ observations. The scatter plots are presented for four separate periods: the pre-euro period (January 1990 to December 1998); the date of the adoption of the euro and prior to the GFC (January 1999 to December 2007); the GFC period (January 2008 to December 2009); and the post-GFC period (January 2010 to May 2017).[2] In

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[1] As Cyprus, Estonia, Latvia, Lithuania, Malta, Slovakia and Slovenia adopted the euro after 2007, they are excluded from the empirical analysis because of the relatively short sample period.

[2] This choice of sub-sample periods is based on a combination of institutional features and events

the case of Belgium, Ireland, Italy, Luxembourg and Portugal, data on equity prices are not available for the first sub-period.

An inspection of the scatter plots in Figure 1 (Austria, Belgium, Finland, France, Germany, Greece) and Figure 2 (Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain) reveals distinct changes in the interrelationships between equity returns in the euro zone and the U.S. across the four periods. For some of the euro zone countries there is a change in the strength of the comovements with the U.S. compared to the pre-euro period, with the scatter plots in the second period revealing a tighter scatter especially for Finland, France and Germany. The effects of the GFC are highlighted by the much tighter scatter plots between U.S. and euro zone equity markets, suggesting even greater global interdependence between European and U.S. equities. For the post-GFC period, as given by the final columns of Figures 1 and 2, there appears to be a reduction in the strength of the comovements between the U.S. and euro zone equity markets with all scatter plots now exhibiting greater dispersion compared to the GFC period.

3   Approach

This section provides the details of constructing a general measure of financial interdependence using entropy theory. An important feature of the approach is that various channels linking asset markets are allowed for including not only second order moments, but also higher order moments such as coskewness, cokurtosis and covolatility. In turn, it is possible to decompose the level of interdependence into these various components and hence identify the size and the direction of alternative sub-measures of interdependence on the overall level of interdependence of asset markets.

3.1   Measuring Interdependence

The main message arising from the bivariate scatter plots of equity returns in Figures 1 and 2 is that it is the joint distribution that determines the strength of the

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in time, as well as econometric methods based on endogenous structural break tests. The structural break tests adopted represent a generalization of the Diebold and Chen (1996) test to a multivariate VAR setting. A likelihood ratio test is used with standard errors based on a Wild paired bootstrap to correct for heteroskedasticity and to preserve the contemporaneous correlation structure in equity returns across asset markets.

Figure 1: Scatter plots of weekly percentage equity returns between selected euro zone countries and the U.S. over four periods of the pre-euro (January 5, 1990 to December 25, 1998), euro (January 1, 1999 to December 28, 2007), the GFC (January 4, 2008 to December 25, 2009), and post-GFC (January 1, 2010 to May 5, 2017) periods respectively. The solid line is the line of best fit.
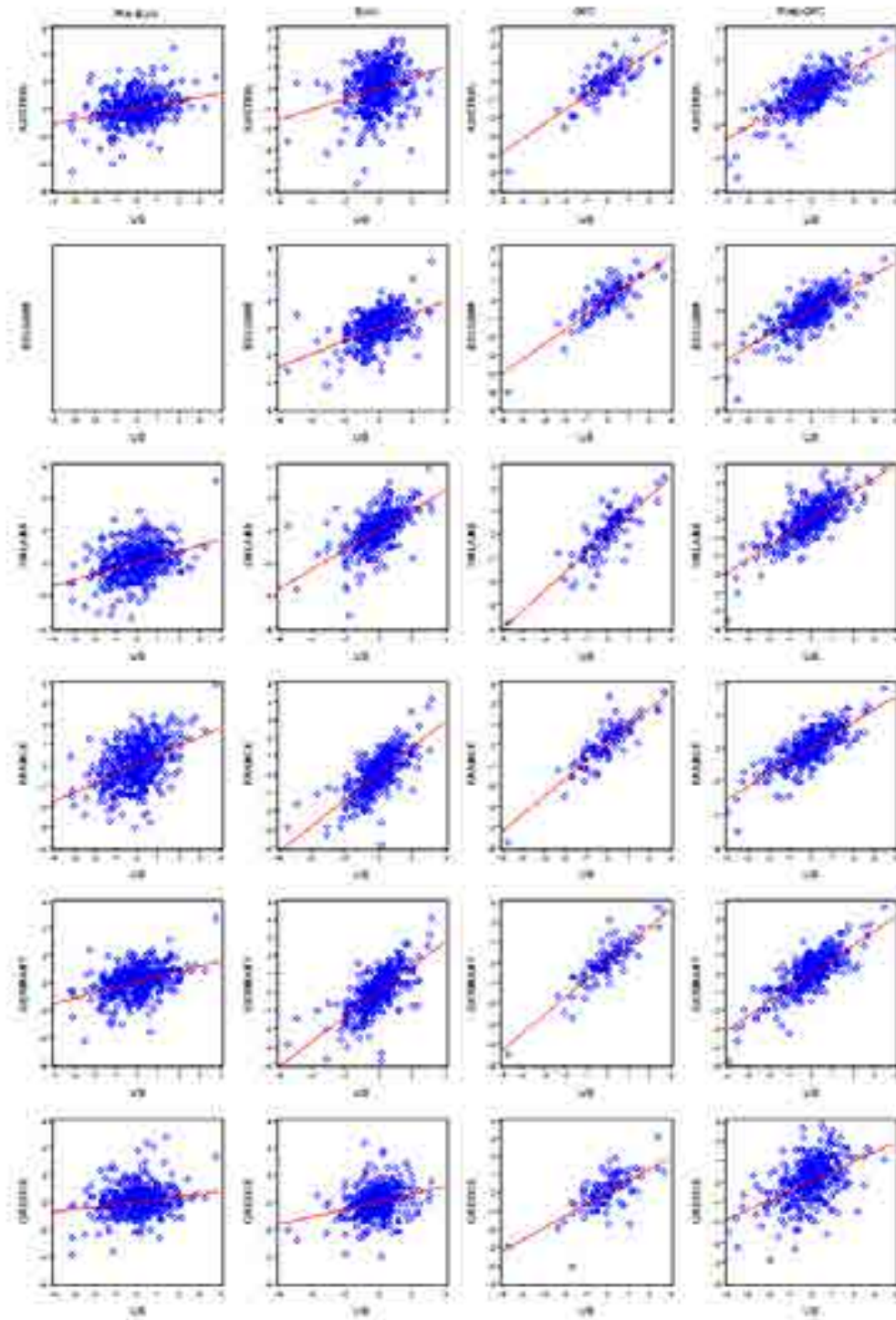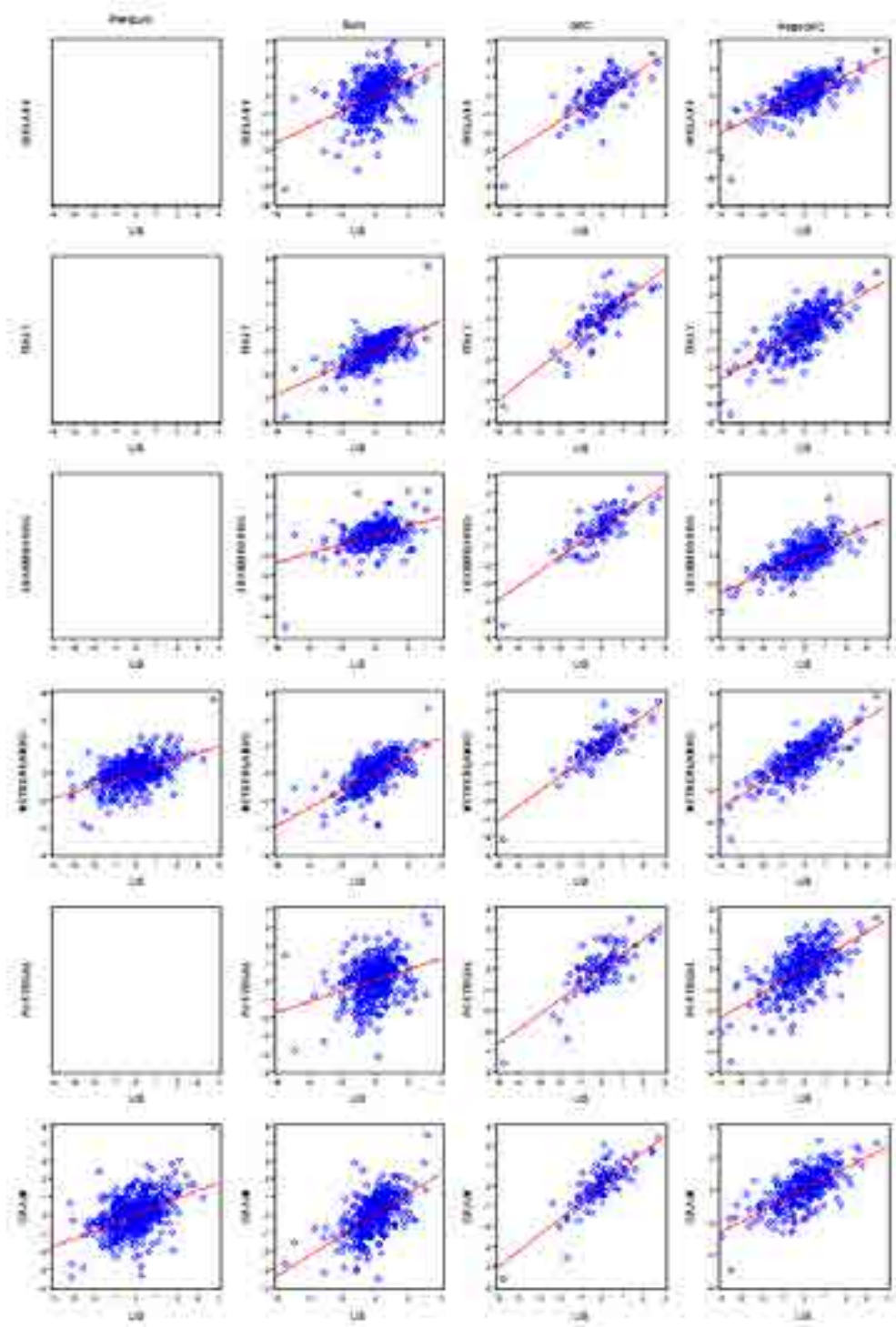
5

Figure 2: Scatter plots of weekly percentage equity returns between selected euro zone countries and the U.S. over four periods of the pre-euro (January 5, 1990 to December 25, 1998), euro (January 1, 1999 to December 28, 2007), the GFC (January 4, 2008 to December 25, 2009), and post-GFC (January 1, 2010 to May 5, 2017) periods respectively. The solid line is the line of best fit.

6

interaction amongst asset markets. In the extreme case of independence where asset markets are segmented, shocks in one market have no clear predictable effects on other asset markets. In the case where asset markets are connected the joint distribution is characterized by regions on the support where joint movements in returns are more likely to occur resulting in the distribution exhibiting higher mass in some regions and lower mass in others. As more and more of the mass of the joint distribution becomes located in a smaller and smaller region of the support of the distribution, asset markets become increasingly more interdependent.

To formalize the interdependence measure, let r_1 and r_2 represent the returns on two asset markets with joint probability distribution $f(r_1, r_2; \Theta)$ with unknown parameter vector Θ. The aim is to choose a measure which increases as asset markets become more dependent and decreases as asset markets become more independent. An appropriate function from information theory is the natural logarithm of the joint probability, $\log f$. For regions of the support of the distribution where there is higher probability of r_1 and r_2 interacting with each other $\log f$ increases, that is it becomes less negative. For those regions where joint events between r_1 and r_2 are less likely the value of the function decreases, that is it becomes more negative. In the extreme case of independence $\log f$ approaches $-\infty$. To construct an overall measure of interdependence the approach is to weight $\log f$ according to the probability that each event occurs. For a continuous joint distribution a formal measure of interdependence is the expected value of the natural logarithm of the distribution

$$\Psi = E\left[\log f(r_1, r_2; \Theta)\right] = \int \int \log\left(f(r_1, r_2; \Theta)\right) f(r_1, r_2; \Theta)\, dr_1 dr_2. \qquad (1)$$

In the extreme case of market segmentation where the joint probability is $f(r_1, r_2; \Theta) = 0$, then $\log(f(r_1, r_2; \Theta))f(r_1, r_2; \Theta) = 0$ is treated as zero. The statistic Ψ also represents the negative of the entropy of a distribution which provides a measure of uncertainty. Smaller values of Ψ represent greater uncertainty as to how returns in different markets respond to shocks, and thus correspond to higher entropy. In the extreme case of independence the asset markets are segmented with Ψ reaching a minimum corresponding to maximum entropy. For increasing values of Ψ there is greater certainty between the connectedness of asset markets resulting in lower entropy.

To derive a general relationship between the interdependence measure in (1) and higher order comoments, a natural and convenient choice is a subordinate distribution

7

from the exponential family (Cobb, Koppstein, and Chen (1983), Lye and Martin (1993), Fry, Martin and Tang (2010)) with the general form

$$f(r_1, r_2; \Theta) = \exp(h - \eta), \tag{2}$$

where $h = h(r_1, r_2; \Theta)$ and $\eta = \eta(\Theta)$ is a normalizing constant defined as

$$\eta = \log \int \int \exp(h) \, dr_1 dr_2, \tag{3}$$

which ensures the probability density integrates to unity, $\int \int f(r_1, r_2; \Theta) \, dr_1 dr_2 = 1$. Upon substituting (2) in (1) yields an alternative form of the interdependence measure

$$\Psi = E[h - \eta] = E[h] - \eta. \tag{4}$$

In choosing the joint distribution of the asset returns and hence an expression for h in (4), the approach is to use the generalized exponential family of distributions which nests a broad range of distributions commonly adopted in empirical finance. In the case where the bivariate generalized normal distribution is chosen as the subordinate distribution the form of h is

$$
\begin{aligned}
h \;=\; & -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2 + \left(\frac{r_2-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{r_1-\mu_1}{\sigma_1}\right)\left(\frac{r_2-\mu_2}{\sigma_2}\right)\right) \\
& +\theta_1\left(\frac{r_1-\mu_1}{\sigma_1}\right)^1\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2 + \theta_2\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\left(\frac{r_2-\mu_2}{\sigma_2}\right)^1 \\
& +\theta_3\left(\frac{r_1-\mu_1}{\sigma_1}\right)^1\left(\frac{r_2-\mu_2}{\sigma_2}\right)^3 + \theta_4\left(\frac{r_1-\mu_1}{\sigma_1}\right)^3\left(\frac{r_2-\mu_2}{\sigma_2}\right)^1 \\
& +\theta_5\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2,
\end{aligned}
\tag{5}
$$

where $\{\mu_1, \sigma_1, \mu_2, \sigma_2, \rho, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$ are the unknown parameters. The first part of the expression represents the bivariate normal distribution which occurs by imposing the restrictions $\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = 0$, where interdependence between r_1 and r_2 is solely controlled by the correlation parameter ρ. The role of higher order comoments is captured by relaxing these restrictions, with coskewness controlled by the parameters θ_1 and θ_2, cokurtosis by the parameters θ_3 and θ_4, and covolatility by the parameter θ_5. It is shown in Appendix A that this choice of h results in $f(r_1, r_2; \Theta)$ in (2) having the optimality property that it maximizes entropy subject to achieving the higher order comoments of coskewness, cokurtosis and covolatility, as well as the second order comoment of correlation.

Using (5) in (4) gives

$$
\begin{aligned}
\Psi = & -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\right]+E\left[\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2\right]\right. \\
& \left.-2\rho E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)\left(\frac{r_2-\mu_2}{\sigma_2}\right)\right]\right) \\
& +\theta_1 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^1\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2\right]+\theta_2 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\left(\frac{r_2-\mu_2}{\sigma_2}\right)^1\right] \quad (6)\\
& +\theta_3 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^1\left(\frac{r_2-\mu_2}{\sigma_2}\right)^3\right]+\theta_4 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^3\left(\frac{r_2-\mu_2}{\sigma_2}\right)^1\right] \\
& +\theta_5 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2\right]-\eta.
\end{aligned}
$$

Defining $E\left[\left(\left(r_1-\mu_1\right)/\sigma_1\right)^2\right]=E\left[\left(\left(r_2-\mu_2\right)/\sigma_2\right)^2\right]=1$ and $E\left[\left(r_1-\mu_1\right)\left(r_2-\mu_2\right)/\sigma_1\sigma_2\right]=\rho$, the first three terms of (6) reduce to

$$
-\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\right]+E\left[\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2\right]-2\rho E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)\left(\frac{r_2-\mu_2}{\sigma_2}\right)\right]\right)=-1,
$$

resulting in (6) simplifying as

$$
\begin{aligned}
\Psi = & \theta_1 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^1\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2\right]+\theta_2 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\left(\frac{r_2-\mu_2}{\sigma_2}\right)^1\right] \\
& +\theta_3 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^1\left(\frac{r_2-\mu_2}{\sigma_2}\right)^3\right]+\theta_4 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^3\left(\frac{r_2-\mu_2}{\sigma_2}\right)^1\right] \quad (7)\\
& +\theta_5 E\left[\left(\frac{r_1-\mu_1}{\sigma_1}\right)^2\left(\frac{r_2-\mu_2}{\sigma_2}\right)^2\right]-(1+\eta).
\end{aligned}
$$

This expression shows that Ψ is a function of the higher order comoments including coskewness, cokurtosis and covolatility. For the special case of bivariate normality $\theta_1=\theta_2=\theta_3=\theta_4=\theta_5=0$, (7) reduces to

$$
\Psi_{BN}=-\left(1+\eta_{BM}\right)=-\left(1+\log\left(2\pi\right)+\frac{1}{2}\log\left(\sigma_1^2\sigma_2^2\left(1-\rho^2\right)\right)\right), \quad (8)
$$

which uses the property of the bivariate normal distribution that an analytical expression is available for the normalizing constant η, in this case given by $\eta_{BN}=\log\left(2\pi\right)+\log\left(\sigma_1^2\sigma_2^2\left(1-\rho^2\right)\right)/2$. Inspection of (8) shows that increases in the (absolute) value of the correlation between and r_1 and r_2, causes an increase in Ψ which represents

an increase in interdependence. This function also has a minimum at $\rho = 0$ given by $-\left(1 + \log\left(2\pi\right) + \ln\left(\sigma_1^2 \sigma_2^2\right)/2\right)$.

Figure 3 gives the simple case of a (standardized) bivariate normal distribution, where $\mu_1 = \mu_2 = 0$ and $\sigma_1^2 = \sigma_2^2 = 1$. Figures 3(a) and 3(b) demonstrate the case of independence with the correlation parameter set at $\rho = 0.0$. The probability contour plot depicting independence in Figure 3(b) shows that equal probabilities are represented by concentric circles. Setting $\rho = 0$ in (8) for independence, together with $\sigma_1^2 = \sigma_2^2 = 1$ because of the standardization, $\Psi_0 = -\left(1 + \log\left(2\pi\right)\right) = -2.8379$. The effect on Ψ by allowing for dependence through positive correlation with $\rho = 0.3$, is given in Figures 3(c) and 3(d). The contour plot in Figure 3(d) shows that the contour probabilities are now ellipsoidal with more of the mass of the distribution located along the main axis of the ellipsoids which has positive slope. This increase in concentration from the case of independence as given in Figures 3(a) and 3(b), now yields a higher value of interdependence equal to

$$\Psi_{BN} = -\left(1 + \ln\left(2\pi\right) + \frac{1}{2}\ln\left(1 - 0.3^2\right)\right) = -2.7907.$$

Figure 4 shows the effects of covolatility and coskewness on the (standardized) generalized bivariate normal distribution with $\rho = 0.3$. Figures 4(a) and 4(b) demonstrate the effects of covolatility on the distribution by setting $\theta_5 = -0.5$ and $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 0$. A comparison of the probability contour plots in Figures 4(b) and 3(d) shows that covolatility tends to have a greater impact on the tails of the distribution where the probability contours take on the shape of a parallelogram. From (7) the effect of covolatility on the entropy measure of interdependence is -2.7586, which is higher than the value of -2.7907 where dependence between r_1 and r_2 is solely through the parameter ρ.[3]

The inclusion of coskewness in addition to covolatility is demonstrated in Figures 4(c) and 4(d) by setting $\theta_2 = -0.6$ and $\theta_5 = -0.5$. Coskewness has the effect of stretching the contours in the positive and negative directions of r_1 in the case of negative values of r_2, while increasing the concentration of the distribution around $r_1 = r_2 = 0.0$. Moreover, the additional coskewness results in an increase in interdependence with Ψ increasing from -2.7586 to -2.7103.

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[3] A bivariate numerical integration procedure is used to evaluate the expectations in (7).
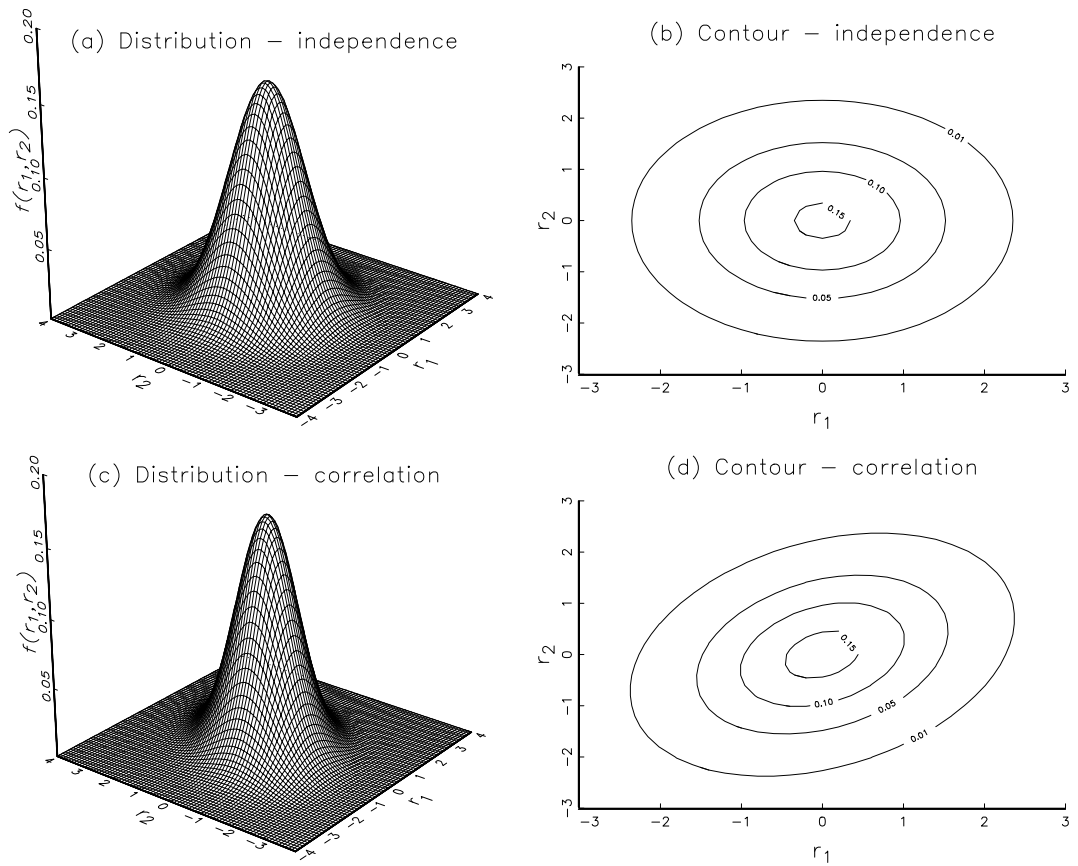
Figure 3: Bivariate (standardized) normal distributions: Independence is given by Figures (a) and (b) assuming a correlation parameter of $\rho = 0.0$. Dependence is given by Figures (c) and (d) assuming a correlation parameter of $\rho = 0.3$.
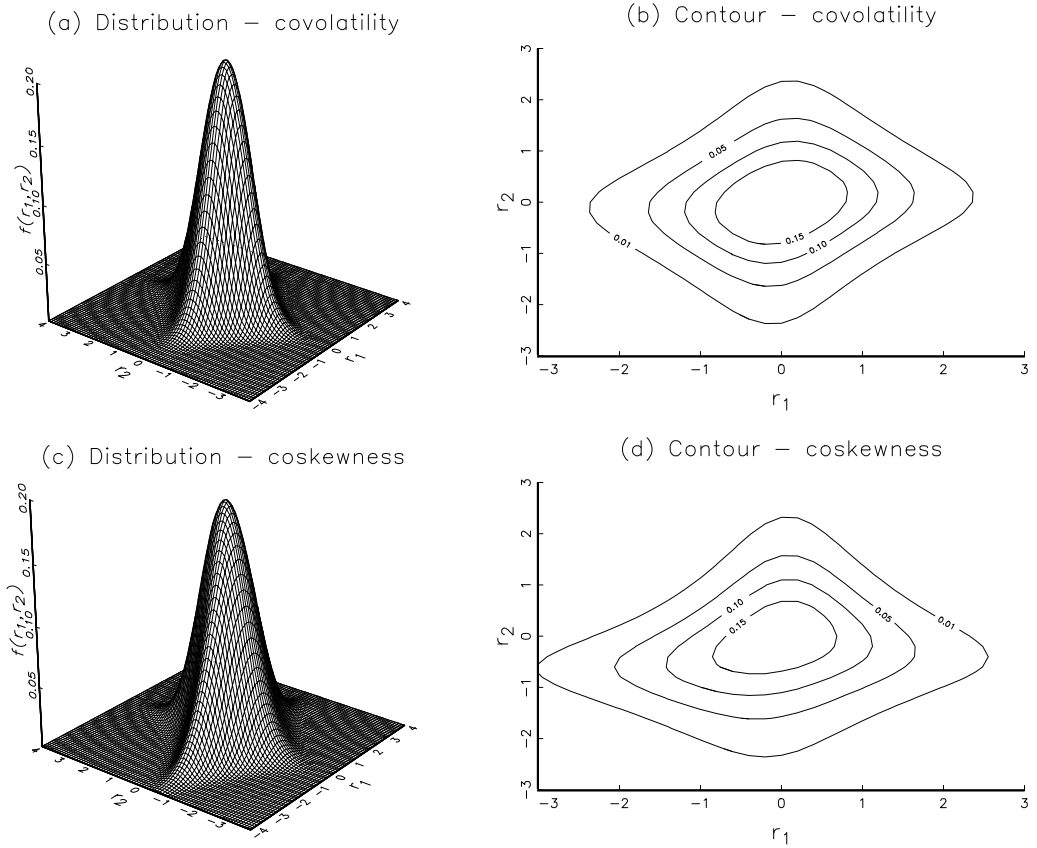
Figure 4: Bivariate generalized (standardized) normal distributions: Covolatility is given by Figures (a) and (b) assuming $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 0$, $\theta_5 = -0.5$, and $\rho = 0.3$. Coskewness is given by Figures (c) and (d) assuming $\theta_2 = -0.6$, $\theta_1 = \theta_3 = \theta_4 = 0$, $\theta_5 = -0.5$ and $\rho = 0.3$.

12

3.2 Implementation

In practice to evaluate Ψ in (4) it is necessary to estimate the unknown parameters Θ, which is achieved by deriving the sample analogue of Ψ. Assuming that the law of large numbers holds an approximation of the expectations $E[h]$ is given by the sample average

$$\frac{1}{T}\sum_{t=1}^{T} h_t(\Theta) \xrightarrow{p} E[h]. \tag{9}$$

This suggests that the sample equivalent measure of interdependence in (4) is

$$\Psi_T(\Theta) = \frac{1}{T}\sum_{t=1}^{T} h_t(\Theta) - \eta(\Theta). \tag{10}$$

The expression on the right hand-side of (10) is effectively the log-likelihood function of the generalized exponential distribution for a sample of $t = 1, 2, \cdots, T$ observations. Evaluating this expression using a consistent estimator $\widehat{\Theta}$ of Θ, the interdependence measure is evaluated as

$$\Psi_T(\widehat{\Theta}) = \frac{1}{T}\sum_{t=1}^{T} h_t(\widehat{\Theta}) - \eta(\widehat{\Theta}), \tag{11}$$

where in the case of the generalized normal distribution in (5) $h_t(\widehat{\Theta})$ is defined as

$$
\begin{aligned}
h_t(\widehat{\Theta}) \;=\; & -\frac{1}{2}\left(\frac{1}{1-\widehat{\rho}^2}\right)\left(\left(\frac{r_{1t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2 + \left(\frac{r_{2t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 - 2\widehat{\rho}\left(\frac{r_{1t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)\right) \\
& + \widehat{\theta}_1\left(\frac{r_{1t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2 + \widehat{\theta}_2\left(\frac{r_{1t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right) \\
& + \widehat{\theta}_3\left(\frac{r_{1t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)\left(\frac{r_{2t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^3 + \widehat{\theta}_4\left(\frac{r_{1t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^3\left(\frac{r_{2t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right) \\
& + \widehat{\theta}_5\left(\frac{r_{1t}-\widehat{\mu}_1}{\widehat{\sigma}_1}\right)^2\left(\frac{r_{2t}-\widehat{\mu}_2}{\widehat{\sigma}_2}\right)^2.
\end{aligned}
\tag{12}
$$

Equations (11) and (12) suggest that to evaluate $\Psi_T(\widehat{\Theta})$, the unknown parameters

$$\Theta = \{\mu_1, \sigma_1, \mu_2, \sigma_2, \rho, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}, \tag{13}$$

are chosen to maximize the log-likelihood function of the bivariate generalized exponential distribution in (10), with the interdependence measure Ψ_T representing the log-likelihood function evaluated at the maximum likelihood estimates. As the log-likelihood of the generalized exponential distribution in (10) is a nonlinear function

of the unknown parameters Θ, an iterative gradient algorithm is needed to compute the maximum likelihood estimates. In the application in Section 5 the GAUSS software MAXLIK Version 10 is used. To reduce the computations the returns data are standardized which reduces the number of unknown parameters in (13) to 6 given by $\{\rho, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$.

3.3 Testing for Independence

The estimator of the entropy measure of interdependence in (11) can be used to generate a test of independence by comparing its unrestricted value with the corresponding measure under the null hypothesis of independence. As already noted above imposing the independence restrictions $\rho = \theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = 0$, an analytical expression for Ψ is available under the null hypothesis. From (7) and (8), imposing these restrictions in the case where the variables are standardized, Ψ under the null hypothesis simply reduces to the constant

$$\Psi_0 = -\left(1 + \ln\left(2\pi\right)\right) = -2.8379. \tag{14}$$

This result suggests that a test of interdependence can be based on the test statistic

$$I = \left(\frac{\Psi_T(\widehat{\Theta}) - \Psi_0}{se(\Psi(\widehat{\Theta}))}\right), \tag{15}$$

where $\Psi_T(\widehat{\Theta})$ is given by (11) with $h_t(\widehat{\Theta})$ defined in (12) in the case of the generalized normal distribution, Ψ_0 is given in (14) and $se(\Psi(\widehat{\Theta}))$ is the standard error of $\Psi(\widehat{\Theta})$. Under the null hypothesis this statistic has an asymptotic normal distribution with zero mean and unit variance.

To generate an expression for the standard error in (15) the delta method is used

$$se(\Psi(\widehat{\Theta})) = \sqrt{D'\Omega D}, \tag{16}$$

where D is a $(N \times 1)$ vector of the derivatives of the entropy measure of interdependence with respect to the parameters Θ evaluated at $\widehat{\Theta}$, and Ω is the variance covariance matrix of $\widehat{\Theta}$ which is obtained directly from the maximum likelihood procedure. The derivatives in D are obtained from

$$\Psi_T = \frac{1}{T} \sum_{t=1}^{T} h_t - (1 + \eta), \tag{17}$$

14

where

$$h_t = -\frac{1}{2}\left(\frac{1}{1-\rho^2}\right)\left(z_{1t}^2 + z_{2t}^2 - 2\rho z_{1t} z_{2t}\right) + \theta_1 z_{1t} z_{2t}^2 + \theta_2 z_{1t}^2 z_{2t} + \theta_3 z_{1t} z_{2t}^3 + \theta_4 z_{1t}^3 z_{2t} + \theta_5 z_{1t}^2 z_{2t}^2,$$

z_{it} represent the standardized variables and

$$\eta = \log \int \int \exp(h)\, dz_1 dz_2, \tag{18}$$

is the normalizing constant. Taking the derivatives of (17) with respect to the unknown parameters in Θ, gives

$$
\begin{aligned}
\frac{\partial \Psi_T}{\partial \rho} &= \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{(\rho^2-1)^2}\left((\rho^2+1)\,z_{1t}z_{2t} - \rho z_{1t}^2 - \rho z_{2t}^2\right)\right) - \frac{\partial \eta}{\partial \rho}\\[2mm]
\frac{\partial \Psi_T}{\partial \theta_1} &= \frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t}^2 - \frac{\partial \eta}{\partial \theta_1}\\[2mm]
\frac{\partial \Psi_T}{\partial \theta_2} &= \frac{1}{T}\sum_{t=1}^{T} z_{1t}^2 z_{2t} - \frac{\partial \eta}{\partial \theta_2}\\[2mm]
\frac{\partial \Psi_T}{\partial \theta_3} &= \frac{1}{T}\sum_{t=1}^{T} z_{1t}z_{2t}^3 - \frac{\partial \eta}{\partial \theta_3}\\[2mm]
\frac{\partial \Psi_T}{\partial \theta_4} &= \frac{1}{T}\sum_{t=1}^{T} z_{1t}^3 z_{2t} - \frac{\partial \eta}{\partial \theta_4}\\[2mm]
\frac{\partial \Psi_T}{\partial \theta_5} &= \frac{1}{T}\sum_{t=1}^{T} z_{1t}^2 z_{2t}^2 - \frac{\partial \eta}{\partial \theta_5}.
\end{aligned}
\tag{19}
$$

Assuming that the orders of integration and differentiation can be reversed the derivatives of η with respect to the parameters are given by

$$\frac{\partial \eta}{\partial \Theta_i} = \frac{\partial}{\partial \Theta_i}\log \int \int \exp(h)\, dz_1 dz_2 = \frac{\int \int \frac{\partial}{\partial \Theta_i}\exp(h)\, dz_1 dz_2}{\int \int \exp(h)\, dz_1 dz_2}, \tag{20}$$

where Θ_i is an element of the parameter vector Θ. Using (20) in (19) gives the following

expressions for the derivatives

$$\frac{\partial \eta}{\partial \rho} = \frac{1}{(\rho^2 - 1)^2} \int \int \left(\left(\rho^2 + 1 \right) z_1 z_2 - \rho z_1^2 - \rho z_2^2 \right) \exp \left(h - \eta \right) dz_1 dz_2$$

$$= \frac{1}{(\rho^2 - 1)^2} E \left(\left(\left(\rho^2 + 1 \right) z_1 z_2 - \rho z_1^2 - \rho z_2^2 \right) \right)$$

$$\frac{\partial \eta}{\partial \theta_1} = \int \int z_1 z_2^2 \exp \left(h - \eta \right) dz_1 dz_2 = E \left(z_1 z_2^2 \right)$$

$$\frac{\partial \eta}{\partial \theta_2} = \int \int z_1^2 z_2 \exp \left(h - \eta \right) dz_1 dz_2 = E \left(z_1^2 z_2 \right)$$

$$\frac{\partial \eta}{\partial \theta_3} = \int \int z_1 z_2^3 \exp \left(h - \eta \right) dz_1 dz_2 = E \left(z_1 z_2^3 \right)$$

$$\frac{\partial \eta}{\partial \theta_4} = \int \int z_1^3 z_2 \exp \left(h - \eta \right) dz_1 dz_2 = E \left(z_1^3 z_2 \right)$$

$$\frac{\partial \eta}{\partial \theta_5} = \int \int z_1^2 z_2^2 \exp \left(h - \eta \right) dz_1 dz_2 = E \left(z_1^2 z_2^2 \right).$$

As the maximum likelihood estimator of Θ is based on solving the gradient expressions in (19), these expressions show that the maximum likelihood procedure applied to the generalized exponential distribution is obtained by finding the estimator which equates the sample and expected moments of the distribution.

4   Simulation Experiments

This section provides the results of some simulation experiments to determine the ability of the entropy interdependence measure given in Section 3 to identify changes in interdependence over time where the data generating processes are taken from models used in empirical finance. The models consist of the bivariate dynamic CAPM of Bekaert and Harvey (1995) and the multi-asset cross-market dispersion model of Solnik and Roulet (2000). To capture changes in interdependence over time the strategy adopted follows Diebold and Yilmaz (2009,2014) whereby dynamic measures of interdependence are constructed by estimating the entropy measure of interdependence using a rolling-window over the sample period. The results presented below are for a single simulation run, with additional simulations reported in Appendix B.[4]

---

[4]In choosing the window width there is a trade-off between having a sufficient number of observations in each subsample to obtain reliable parameter estimates, and not choosing too wide a window that smooths out the effects of events on the interdependence measure. In the simulation experiments a window width of 130 is adopted to satisfy these requirements, although other window widths give very similar results.

4.1 Bekaert-Harvey Model of Integration

Bekaert and Harvey (1995) specify a dynamic CAPM that is widely used in the literature to identify changes in integration over time. Letting r_{it} represent the excess returns on a portfolio in country i, and r_{wt} the excess returns on a world portfolio, the model is specified as

$$\begin{aligned} r_{it} &= \phi_t \lambda_{wt} \sigma_{iwt} + (1 - \phi_t) \lambda_{it} \sigma_{it}^2 + u_{it} \\ r_{wt} &= \lambda_{wt} \sigma_{wt}^2 + u_{wt}, \end{aligned} \qquad (21)$$

where $0 \leq \phi_t \leq 1$ is the time-varying measure of integration. In the extreme case of full global market integration $\phi_t = 1$, with the country risk premium determined by the product of the world price of covariance risk (λ_{wt}) and the covariance between country and world excess returns (σ_{iwt}). At the other extreme of market segmentation, $\phi_t = 0$ with the risk premium determined by the product of the local price of risk (λ_{it}) and the local excess returns variance (σ_{it}^2). The disturbances u_{it} and u_{wt} are normally distributed with zero means and conditional covariance matrix based on the constant correlation bivariate GARCH specification. Time-variation in excess returns comes from four sources: the integration measure (ϕ_t), risk prices $(\lambda_{wt}, \lambda_{it})$, risk quantities $(\sigma_{iwt}, \sigma_{it}^2, \sigma_{wt}^2)$ and idiosyncratic risks (u_{it}, u_{wt}).

The model in (21) is simulated by specifying the time-varying measure of integration ϕ_t, as a random walk with positive drift. The positive drift parameter ensures that ϕ_t trends upwards resulting in a movement from market segmentation to global integration. The market prices of risk λ_{it} and λ_{wt}, are assumed to be stochastic with a mean of 2.1. The risk quantities are determined using a bivariate GARCH constant correlation model with σ_{it}^2 and σ_{wt}^2 having GARCH specifications of the general form $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$, with parameters $\alpha_0 = 0.05, \alpha_1 = 0.05, \beta_1 = 0.9$ for both asset returns, and correlation parameters of $\rho = 0.0$ (market segmentation) and $\rho = 0.7$ (market interdependence). The conditional variances and covariances in (21) represent weighted averages of the market segmentation and market interdependence conditional covariance matrices using the time-varying integration parameter ϕ_t.

The results of simulating the Bekaert-Harvey model in (21) for $T = 2000$ observations are presented in Figure 5. The two graphs in the first column give the simulated world returns (r_{wt}) and the country returns (r_{it}). The top right panel gives the time-varying integration parameter ϕ_t. In the first part of the sample there is near market

17

segmentation with ϕ_t close to zero. In the second half of the sample ϕ_t trends upwards towards unity as the country asset market becomes more globally integrated.

The entropy based interdependence measure Ψ_t, is given in the bottom right panel using a rolling window of length 130 periods applied to the simulated returns, r_{wt} and r_{it}. Through the lens of r_{wt} and r_{it}, Ψ_t tracks the general changes in interdependence over time as the model switches from market segmentation to global integration, even though ϕ_t is a latent variable and hence not directly observed. The value of Ψ_t is relatively low in the initial sample period which is consistent with the local asset market being nearly perfectly segmented. After which, Ψ_t trends upwards reflecting the increasing role of global factors in determining the country's risk premium. The overall correlation between ϕ_t and Ψ_t is 0.88.

4.2 Solnik-Roulet Model of Cross-sectional Dispersion

Solnik and Roulet (2000) identify changes in financial interdependence over time using a multivariate measure of dispersion based on the cross-market standard deviation of returns at each point in time (σ_t). Falls in σ_t represent lower cross-sectional dispersion in asset markets and increasing financial interdependence, whereas increases in σ_t represent greater dispersion in cross-sectional returns and a movement towards independence.

To simulate the Solnik-Roulet model the approach is to draw $N = 20$ cross-market returns at each point in time t from a multivariate normal distribution with an equicorrelation matrix at each point in time, with the correlations ρ_t changing over time based on a random walk. The simulated returns are given in Figure 6 for a sample of size $T = 2000$.

A plot of the time-varying standard deviation σ_t is presented in the first panel of Figure 7. Following Yu, Fung and Tam (2010), σ_t is transformed using a Hodrick-Prescott filter with a smoothing parameter of $\gamma = 130$. For the first third of the sample the standard deviation is relatively high suggesting relatively low levels of interdependence amongst the $N = 20$ asset markets. This is followed by a period of increasing financial interdependence with σ_t falling in the middle of the sample after which there is a deterioration of interdependence with σ_t increasing again. For the remainder of the sample σ_t tends to trend downwards corresponding to periods of increased inter-
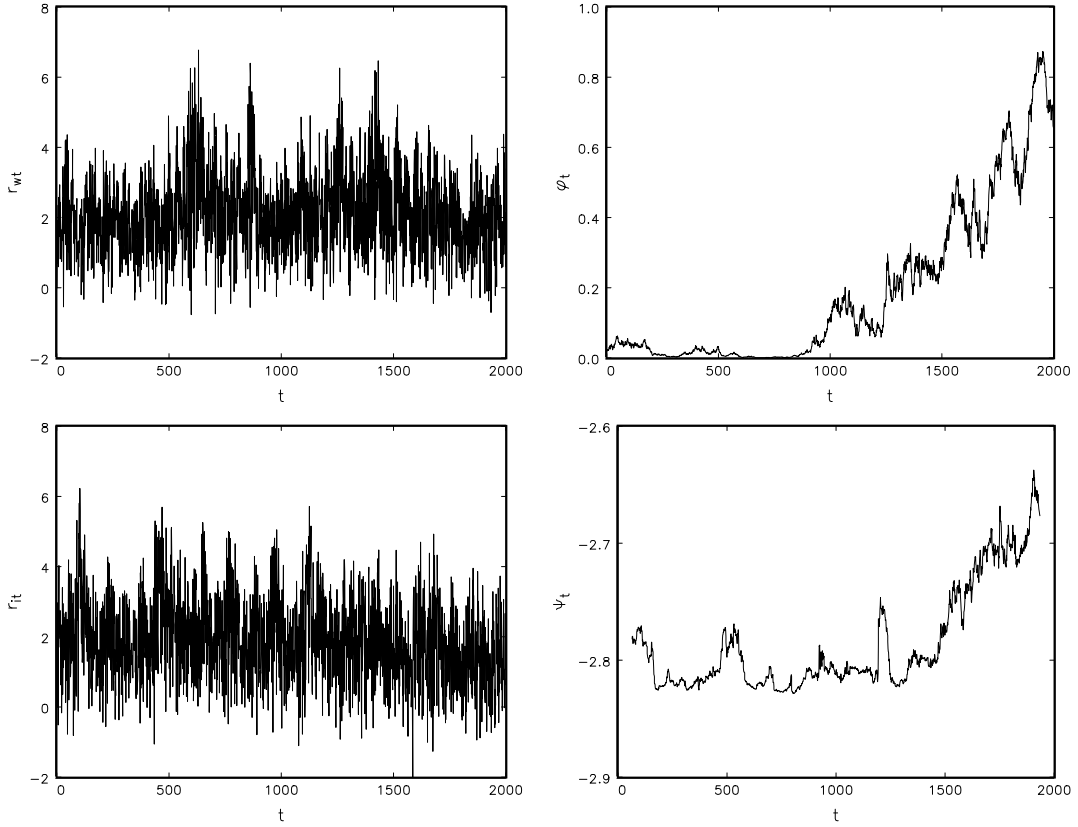
Figure 5: Simulation results of the Bekaert-Harvey model in (21) based on a sample size of $T = 2000$. The first column gives the simulated returns for the world (r_{wt}) and the country (r_{it}). The top panel of the second column gives the measure of financial integration (ϕ_t) with $\phi_t = 1$ representing global market integration and $\phi_t = 0$ representing market segmentation. The bottom panel of the second column gives the interdependence measure Ψ_t applied to the simulated returns with a data window of 130 observations.
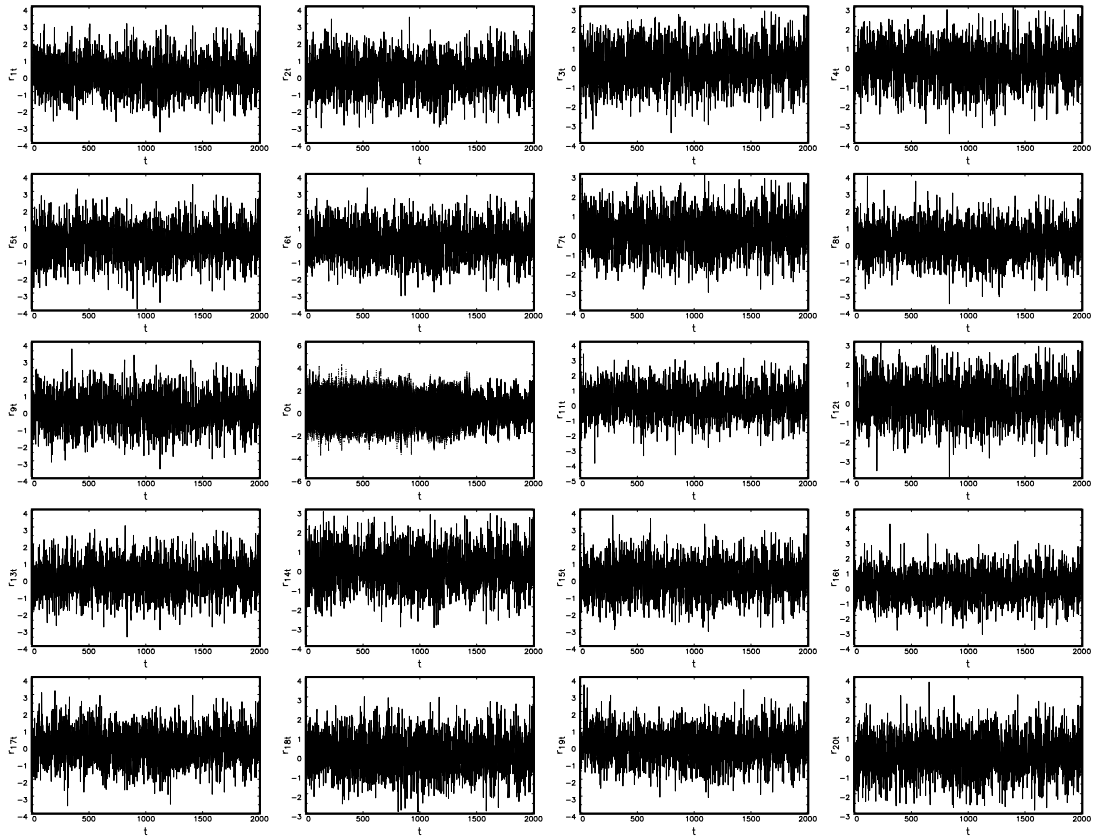
Figure 6: Simulated returns of the Solnik-Roulet model based on a sample size of $T = 2000$ with $N = 20$ asset markets.

dependence.

The interdependence measure Ψ_t is calculated using the $N = 20$ simulated time series of asset returns in Figure 6 with a data window of 130 periods. The interdependence measure Ψ_t is computed for $N - 1$ pairs of asset returns with the Nth market arbitrarily taken as the numeraire asset, with the overall estimate of interdependence obtained by aggregating the log-likelihood values for each pair of assets. The results for Ψ_t are presented in the second panel of Figure 7. Inspection of Figure 7 shows that Ψ_t closely mirrors the movements in the Solnik-Roulet measure of dispersion given by σ_t, producing a very high correlation between σ_t and Ψ_t of 0.97.
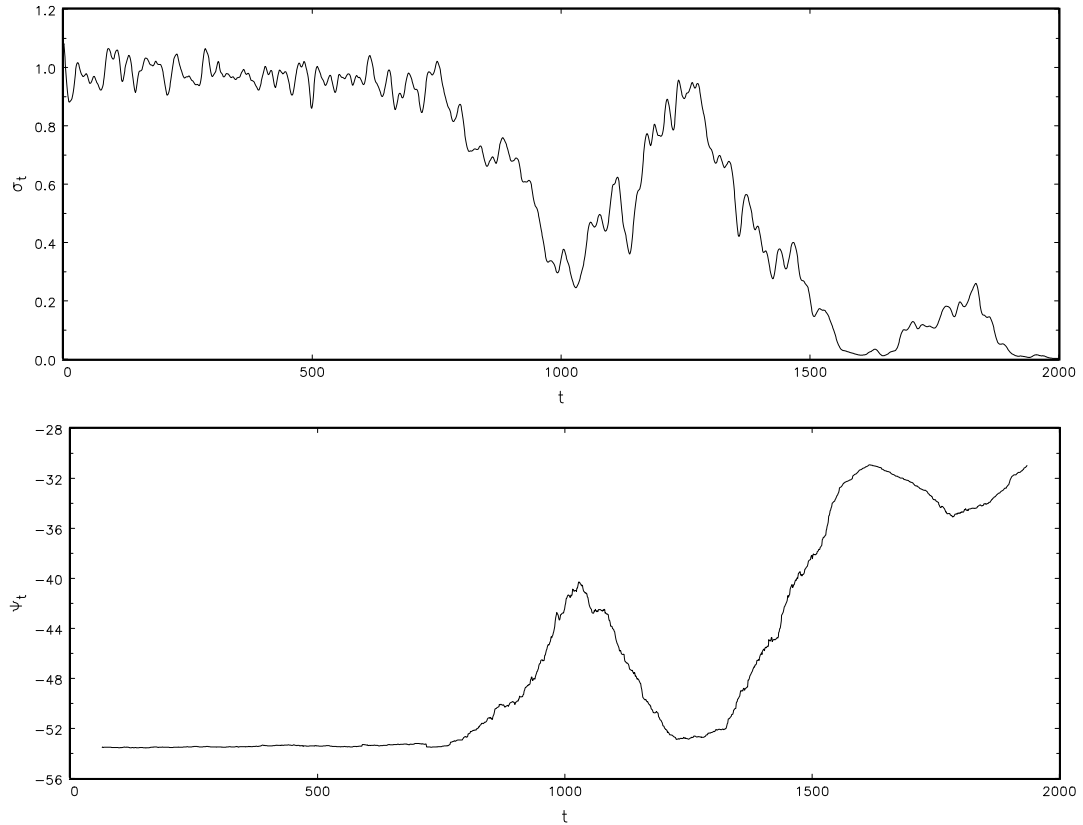
Figure 7: Simulation results of the Solnik-Roulet model based on a sample size of $T = 2000$ with $N = 20$ asset markets. The first panel gives the measure of financial interdependence (σ_t) with falls in σ_t representing increasing financial interdependence and increases in σ_t representing movements towards independence. The second panel is the interdependence measure Ψ_t applied to the simulated country and global returns with a data window of 130 observations.

5 Application to the Euro Zone

The entropy based measure of financial interdependence in (11) is applied to examine changes in comoments of equity markets in the euro zone both globally and regionally. Previous research has used a broad array of techniques to model the various aspects of interdependence of European equity markets within Europe. Models of general European interdependence examine asymmetric volatility spillovers (Baele (2005)), time varying copulas (Bartram, Taylor and Wang (2007)), factor models of international equity market interdependence (Bekaert, Hodrick, and Zhang (2009)), tail dependence using coexceedance probabilities (Beine, Cosma, and Vermeulen (2010)), and short and long term interdependence using wavelets and copulas (Shahzad, Kumar, Ali and Ameer (2016)). Models of European equity market integration use GARCH models (Fratzscher (2002), Kim, Moshirian and Wu (2005)) and factor models (Berger and Pozzi (2013)). Papers examining contagion for Europe are based on Granger causality and vector error correction models (Gentile and Giordano (2013)), sectoral factor models (Bekaert, Ehrmann, Fratzscher and Mehl (2014)), and tests of changes in higher order comoments (Fry-McKibbin, Hsiao and Martin (2017), Chan, Fry-McKibbin and Hsiao (2017)).

5.1 Data

The weekly percentage log-returns on equities from 1990 to 2017 for the same euro zone countries introduced in Section 2 are presented in Figure 8. Illustrative summary statistics are given in Table 1 for the four periods: pre-euro (1990-1998), euro (1999-2007), the GFC (2008-2009) and the post-GFC (2010-2017). The statistics are presented only for the countries with data for the entire duration of the sample period.[5]

For the pre-euro and euro periods all countries experience positive returns in equities apart from Austria in the 1990-98 period. This contrasts with the GFC period where average returns are all negative, which return to being positive during the post-GFC period with the exception of Greece and to a lesser extent Austria and Spain. The effect of the GFC on volatility in Euro zone equity returns as well as in the U.S. is highlighted by the increase in the standard deviation compared to pre-GFC levels.

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[5]Statistics for the missing countries are available on request for the periods when their data does become available.
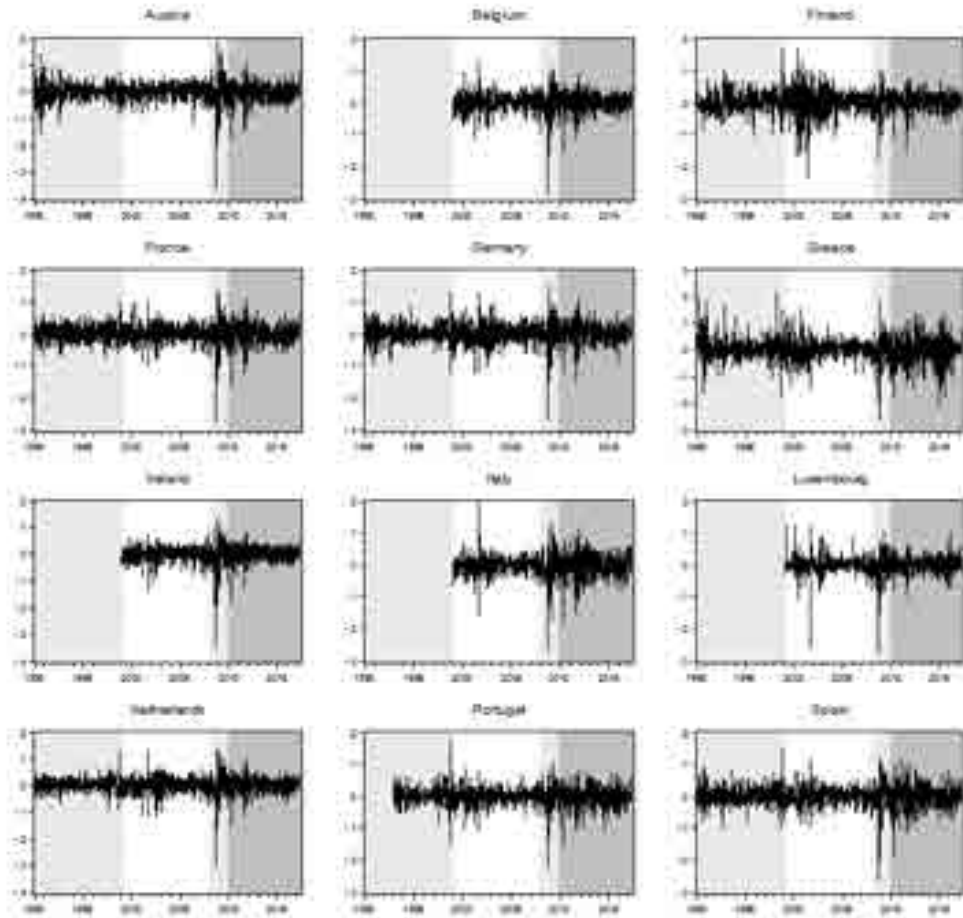
Figure 8: Eurozone weekly equity returns, 5 January 1990 - May 5, 2017. The shaded areas represent the pre-euro (January 5, 1990 to December 25, 1998), euro (January 1, 1999 to December 28, 2007), GFC (January 4, 2008 to December 25, 2009) and post-GFC (January 1, 2010 to May 5, 2017) periods respectively.

There are also increases in kurtosis for the same period with the exception of Finland. During the euro period of 1999 to 2007, equity returns are negatively skewed which become even more skewed during the GFC period, while softening in the post-GFC period.

Table 2 provides comoment statistics between world equity returns as measured by the U.S. equity market, and European equity returns for the same periods. Most European countries with few exceptions experience increases in comoments with the world between the pre and post Euro periods, further increasing during the GFC and softening in the post GFC period. This is true of correlations, covolatility and

23

Table 1:

Summary statistics of a selection of weekly euro zone and U.S. percentage equity returns over four periods of the pre-euro (January 5, 1990 to December 25, 1998), euro (January 1, 1999 to December 28, 2007), the GFC (January 4, 2008 to December 25, 2009), and post-GFC (January 1, 2010 to May 5, 2017) periods respectively. The European equity returns are selected based on availability of data for the entire sample period.

Country	Mean	Min	Max	Std dev	Skewness	Kurtosis
Pre-euro: Jan 12, 1990 to Dec 25, 1998						
Austria	-0.035	-15.611	14.458	3.289	-0.203	5.742
Finland	0.280	-10.877	16.533	3.237	0.161	4.557
France	0.147	-7.811	10.224	2.602	0.001	3.175
Germany	0.215	-11.956	13.359	2.765	-0.233	5.197
Greece	0.356	-17.838	21.541	4.476	0.549	6.339
Netherlands	0.295	-9.461	12.746	2.334	-0.073	5.466
Spain	0.197	-10.269	14.716	2.976	0.088	4.507
U.S.	0.267	-5.324	7.062	1.770	0.001	3.674
Euro: Jan 15, 1999 to Dec 28, 2007						
Austria	0.345	-12.378	6.712	2.687	-0.686	4.455
Finland	0.138	-23.517	16.843	4.429	-0.670	5.944
France	0.112	-10.505	11.056	2.705	-0.232	4.426
Germany	0.137	-12.298	12.909	3.200	-0.335	4.455
Greece	0.167	-14.716	15.687	3.628	-0.065	5.059
Netherlands	0.034	-11.696	13.604	2.905	-0.543	5.322
Spain	0.132	-9.530	11.845	2.682	-0.285	4.246
U.S.	0.032	-12.330	7.492	2.309	-0.586	6.241
GFC: Jan 4, 2008 to Dec 25, 2009						
Austria	-0.606	-36.629	18.534	7.113	-1.227	8.635
Finland	-0.578	-17.976	10.382	4.542	-0.608	4.291
France	-0.373	-27.545	13.739	5.612	-1.034	7.352
Germany	-0.316	-26.841	15.289	5.811	-0.890	6.906
Greece	-0.839	-26.167	17.907	6.134	-0.711	5.588
Netherlands	-0.442	-31.249	13.782	5.868	-1.248	9.137
Spain	-0.253	-26.321	12.408	5.585	-1.268	7.587
U.S.	-0.261	-20.084	11.356	4.237	-0.708	6.983
Post-GFC: Jan 1, 2010 to May 5, 2017						
Austria	-0.012	-18.033	10.984	3.414	-0.880	6.242
Finland	0.109	-15.232	7.599	2.737	-0.742	6.045
France	0.016	-16.153	11.379	3.139	-0.628	5.621
Germany	0.128	-14.755	11.308	3.077	-0.554	5.337
Greece	-0.349	-21.227	13.982	5.285	-0.364	3.930
Netherlands	0.053	-14.572	10.328	2.792	-0.626	5.706
Spain	-0.088	-19.196	11.491	3.823	-0.444	4.661
U.S.	0.197	-7.460	7.128	1.964	-0.447	4.734

cokurtosis. In the case of coskewness, it is negative between all Eurozone countries and the world prior to the GFC, becoming more negative during the GFC period, and decreasing thereafter in the post-GFC period, but not necessarily to pre-GFC levels.

5.2 Global Interdependence

Table 3 provides the global interdependence results based on (11) for Europe as well as for the euro zone country equity markets, with the U.S. chosen as the benchmark global equity market. The results are presented for the pre-euro, euro, GFC and post-GFC periods as well as for the total period. In reporting the interdependence measure the values presented are with respect to the measure of independence Ψ_0 in (14). Also given in the table are the standard errors calculated using the delta method with the ratio of the two representing a t-statistic which under the null hypothesis of independence is asymptotically distributed as $N(0,1)$.

The results given in Table 3 show that Europe operated independently of global equity markets prior to the start of the euro. This changes with the introduction of the euro with global interdependence increasing four-fold from 0.7 to 2.9. Inspection of the individual country results for this period reveal that France and Germany play the major roles in achieving greater global interdependence. This trend increases during the GFC with European equity markets becoming even more globally interconnected with $\Psi(\widehat{\Theta}) - \Psi_0$ for Europe more than doubling from 2.9 to 7.2. The effects of the GFC are widespread with nearly all euro zone countries showing evidence of global interdependence, with the exceptions being Greece and Italy. There is a fall in global interdependence post-GFC for Europe to levels that are nonetheless still higher than they were prior to the GFC with $\Psi(\widehat{\Theta}) - \Psi_0$ decreasing from 7.2 to 4.7. This reduction in equity market interdependence is largely due to Greece, Ireland, Portugal and Spain, and to a lesser extent Luxembourg. As these countries were some of the most affected countries of the crisis this result is consistent with expectations regarding these markets operating independently of global equity markets.[6]

Figure 9 provides an even more refined analysis of changes in global interdependence over time by computing $\Psi(\widehat{\Theta}) - \Psi_0$ over a rolling window for those countries having

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[6]Greece, Portugal and Spain all negotiated financial support packages from the IMF, the EU or a combination of both beginning in 2010 and implemented austerity measures during the post GFC period.

Table 2:

Comoment statistics between weekly world (w) and selected euro zone equity returns (i) over four periods of the pre-euro (January 5, 1990 to December 25, 1998), euro (January 1, 1999 to December 28, 2007), the GFC (January 4, 2008 to December 25, 2009), and post-GFC (January 1, 2010 to May 5, 2017) periods respectively. World equity returns are computed using U.S. equity log-returns.

Country	Correlation (r_w^1, r_i^1)	Coskewness (r_w^1, r_i^2)	(r_w^2, r_i^1)	Covolatility (r_w^2, r_i^2)	Cokurtosis (r_w^1, r_i^3)	(r_w^3, r_i^1)
Pre-euro: Jan 12, 1990 to Dec 25, 1998						
Austria	0.283	-0.188	-0.030	1.821	1.904	1.310
Finland	0.355	0.041	0.056	1.881	1.994	1.560
France	0.451	0.157	0.153	1.841	1.577	1.910
Germany	0.394	-0.022	0.077	2.103	2.254	1.756
Greece	0.186	-0.042	-0.050	1.709	1.681	1.174
Netherlands	0.483	0.068	0.085	2.501	3.047	2.135
Spain	0.445	0.207	0.175	2.381	2.362	2.070
Euro: Jan 15, 1999 to Dec 28, 2007						
Austria	0.262	-0.203	-0.289	1.597	1.348	1.393
Finland	0.604	-0.251	-0.130	2.662	3.090	2.657
France	0.720	-0.106	-0.269	3.061	3.009	3.728
Germany	0.693	-0.108	-0.270	2.969	2.847	3.602
Greece	0.281	-0.190	-0.353	1.824	1.072	2.111
Netherlands	0.662	-0.213	-0.327	2.939	2.974	3.518
Spain	0.574	-0.162	-0.276	2.476	2.318	3.099
GFC: Jan 4, 2008 to Dec 25, 2009						
Austria	0.789	-1.176	-0.979	7.030	7.570	6.736
Finland	0.814	-0.579	-0.631	4.736	4.186	5.591
France	0.857	-0.984	-0.856	6.595	6.726	6.650
Germany	0.867	-0.850	-0.781	6.401	6.427	6.568
Greece	0.655	-0.433	-0.563	3.675	3.556	4.611
Netherlands	0.845	-1.144	-0.944	7.369	7.986	6.993
Spain	0.820	-1.068	-0.872	6.252	6.484	6.402
Post-GFC: Jan 1, 2010 to May 5, 2017						
Austria	0.718	-0.659	-0.514	3.989	4.608	3.847
Finland	0.754	-0.647	-0.515	4.081	4.586	4.001
France	0.784	-0.444	-0.403	3.896	4.323	3.944
Germany	0.786	-0.465	-0.443	4.040	4.359	4.050
Greece	0.473	-0.245	-0.353	1.948	1.830	2.230
Netherlands	0.802	-0.466	-0.435	4.134	4.554	4.105
Spain	0.667	-0.279	-0.295	2.991	3.195	3.217

Note: The comoment statistics between $r_{w,t}^m$ and $r_{i,t}^n$ are computed as $T^{-1} \sum_{t=1}^{T} z_{w,t}^m z_{i,t}^n$, where $z_{w,t} = \left(r_{w,t} - \widehat{\mu}_w \right) / \widehat{\sigma}_w$ and $z_{i,t} = \left(r_{i,t} - \widehat{\mu}_i \right) / \widehat{\sigma}_i$ are respectively the standardized returns for the World and Europe.

Table 3:

Global weekly interdependence measure relative to independence $(\Psi(\widehat{\Theta}) - \Psi_0)$ for the euro zone countries for selected sample periods. Asymptotic standard errors are in parentheses.

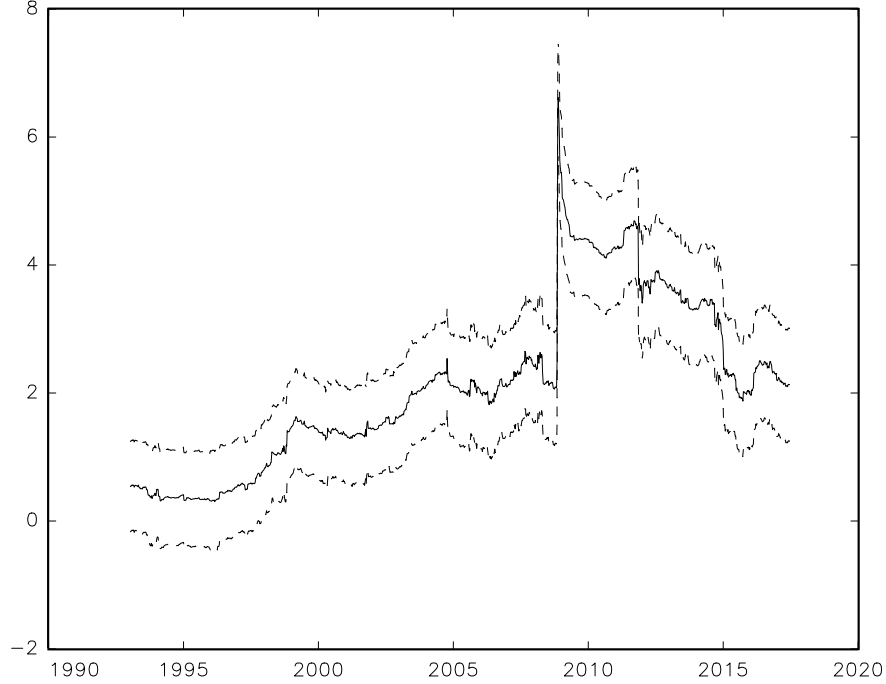| Region /Country	Pre-euro (1990-1998)	Euro (1999-2007)	GFC (2008-2009)	Post-GFC (2010-2017)	Total (1990-2017)
Europe	0.7178	2.9020	7.2404	4.6667	4.5760
	(0.6688)	(0.6217)	(1.2766)	(0.7077)	(0.8629)
Austria	0.0672	0.0578	0.5797	0.3998	0.4599
	(0.1455)	(0.1820)	(0.2737)	(0.1980)	(0.5946)
Belgium	n.a.	0.1626	0.6481	0.4361	0.4231
		(0.1951)	(0.2856)	(0.2046)	(0.2523)
Finland	0.0830	0.2482	0.6037	0.4561	0.2965
	(0.1474)	(0.2404)	(0.2986)	(0.2068)	(0.1925)
France	0.1236	0.4016	0.7365	0.5063	0.4347
	(0.7449)	(0.2412)	(0.3114)	(0.2046)	(0.2201)
Germany	0.1018	0.3567	0.7624	0.5032	0.4092
	(0.2047)	(0.2350)	(0.3002)	(0.2006)	(0.2050)
Greece	0.0565	0.0752	0.3475	0.1450	0.2039
	(0.1838)	(0.1472)	(0.3078)	(0.1910)	(0.3110)
Italy	n.a.	0.1435	0.4817	0.4168	0.4414
		(0.2956)	(0.3124)	(0.2122)	(0.4344)
Ireland	n.a.	0.3510	0.6215	0.3476	0.4228
		(0.2607)	(0.2904)	(0.1933)	(0.2239)
Luxembourg	n.a.	0.4807	0.6169	0.3562	0.4244
		(0.7282)	(0.2781)	(0.2040)	(0.3808)
Netherlands	0.1570	0.3199	0.7198	0.5416	0.4755
	(0.6060)	(0.2404)	(0.3041)	(0.2080)	(0.2487)
Portugal	n.a.	0.0785	0.4902	0.2467	0.2628
		(0.1528)	(0.2676)	(0.1937)	(0.4085)
Spain	0.1287	0.2262	0.6323	0.3114	0.3217
	(0.1679)	(0.2166)	(0.2959)	(0.8335)	(0.6061)

Figure 9: Time-varying estimates of global interdependence for the euro zone countries using a rolling window of 3 years.

data extending over the full sample period. The window width is set at 3 years with 95% confidence intervals given by the dashed lines. Additional results presented in Appendix C show that choosing window widths of 2 years and 4 years does not change the overall qualitative results regarding the general changes in interdependence over time. These results support the sub-period analysis presented in Table 3 with equity markets in euro zone countries becoming increasingly interdependent with global equity markets over time from the start of the euro. The effects of the GFC are also highlighted with the big increase in $\Psi(\widehat{\Theta}) - \Psi_0$, followed by the fall in global interdependence by May 2017 to levels just prior to the GFC occurring.

Table 4 provides a breakdown of the contributions of the comoments, expressed as a percentage of the total, to the changes in global equity market interdependence over time identified in Table 3 and Figure 9. While the covariance is an important contributor to the global interdependence measure, it is the changes in the contributions of the higher order moments that are important in identifying the channels affecting global interdependence. In particular, the increase in global interdependence from the

28

Table 4:

Contribution of the weekly global interdependence measure components for the euro zone, expressed as a percentage, for selected sample periods.

Dependence Measure	Pre-euro (1990-1998)	Euro (1999-2007)	GFC (2008-2009)	Post-GFC (2010-2017)	Total (1990-2017)
Covariance	80.2350	80.1324	95.7959	95.5665	74.3012
Coskewness	1.2002	1.4475	1.4640	0.8590	0.6559
Cokurtosis	30.9475	58.7543	-14.0438	23.7251	63.6793
Covolatility	-12.3827	-40.3342	16.7838	-20.1507	-38.6364

start of the euro is primarily the result of cokurtosis with its contribution nearly doubling during this period, while the contribution of the covariance remains constant. Covolatility provides an offsetting force during this period implying that volatility in European and global equity markets became less dependent on each other. This all changes during the GFC where volatilities in European and global equity markets become more interconnected with the contribution of covolatility becoming positive. This reversal also occurs for cokurtosis which has a dampening effect on interdependence during the GFC. For the post-GFC period the comoments switch back to their pre-GFC roles with the even moments of covariance and cokurtosis having a positive effect on global interdependence and colatility having a dampening effect. Finally, the decompositions presented in Table 4 reveal that coskewess plays a minor role in determining the strength of the relationship between European and global equity markets.

The analysis presented so far is based on identifying contemporaneous relationships between European and global equity markets. To allow for potential dynamic interactions the entropy measure of interdependence is presented in Table 5 where a lead (lag) signifies that the U.S. leads (lags) Europe. The empirical results provide some evidence that global equity markets lead Europe in aggregate, especially during the euro and post-GFC periods, whereas during the GFC the relationship is contemporaneous. From the perspective of the individual European countries no strong dynamic linkages are identified.

Table 5:

Dynamic weekly global interdependence measure $(\Psi(\widehat{\Theta}) - \Psi_0)$ for the euro zone
countries for selected sample periods. Asymptotic standard errors are in parentheses.
A lead (lag) represents the U.S. leads (lags) Europe.

Region /Country	Pre-euro (1990-1998)		Euro (1999-2007)		GFC (2008-2009)		Post-GFC (2010-2017)		Total (1990-2017)	
	Lead	Lag	Lead	Lag	Lead	Lag	Lead	Lag	Lead	Lag
Europe	0.19	0.17	0.75	0.92	1.93	2.03	0.50	0.44	1.54	1.48
	(0.17)	(0.91)	(0.42)	(2.49)	(1.59)	(1.71)	(0.27)	(1.52)	(0.64)	(1.60)
Austria	0.05	0.03	0.03	0.03	0.16	0.16	0.04	0.03	0.17	0.14
	(0.14)	(0.21)	(0.21)	(0.22)	(0.43)	(0.61)	(0.25)	(0.37)	(0.50)	(0.53)
Belgium	n.a.	n.a.	0.07	0.05	0.18	0.19	0.06	0.04	0.15	0.14
			(0.14)	(0.26)	(0.38)	(0.71)	(0.29)	(0.25)	(0.48)	(0.53)
Finland	0.02	0.03	0.03	0.02	0.14	0.10	0.08	0.03	0.06	0.03
	(0.25)	(0.22)	(0.27)	(0.38)	(0.46)	(0.45)	(0.35)	(0.25)	(0.33)	(0.20)
France	0.02	0.02	0.06	0.06	0.17	0.16	0.03	0.04	0.14	0.11
	(0.25)	(0.74)	(0.28)	(0.27)	(0.50)	(0.65)	(0.22)	(0.35)	(0.43)	(0.39)
Germany	0.02	0.03	0.06	0.06	0.14	0.17	0.03	0.03	0.11	0.11
	(0.37)	(0.85)	(0.19)	(0.30)	(0.48)	(0.51)	(0.25)	(0.27)	(0.24)	(0.34)
Greece	0.02	0.02	0.03	0.05	0.13	0.18	0.02	0.03	0.05	0.09
	(0.15)	(0.35)	(0.31)	(0.18)	(0.32)	(0.25)	(0.16)	(0.25)	(0.18)	(0.21)
Italy	n.a.	n.a.	0.03	0.07	0.16	0.16	0.04	0.04	0.27	0.16
			(0.17)	(0.20)	(0.41)	(0.42)	(0.25)	(0.29)	(0.59)	(1.10)
Ireland	n.a.	n.a.	0.18	0.09	0.17	0.15	0.07	0.04	0.14	0.13
			(0.55)	(0.27)	(0.36)	(1.04)	(0.36)	(0.61)	(0.27)	(0.35)
Luxembourg	n.a.	n.a.	0.08	0.33	0.21	0.20	0.03	0.04	0.12	0.22
			(0.48)	(1.33)	(0.41)	(0.85)	(0.27)	(0.24)	(0.30)	(0.93)
Netherlands	0.03	0.02	0.07	0.06	0.16	0.20	0.04	0.04	0.16	0.14
	(0.31)	(0.19)	(0.16)	(0.23)	(0.45)	(0.80)	(0.22)	(0.38)	(0.55)	(0.52)
Portugal	n.a.	n.a.	0.05	0.05	0.16	0.20	0.03	0.03	0.09	0.12
			(0.16)	(0.23)	(0.45)	(0.42)	(0.19)	(0.39)	(0.34)	(0.33)
Spain	0.03	0.02	0.06	0.05	0.14	0.16	0.03	0.04	0.10	0.08
	(0.30)	(0.19)	(0.20)	(0.28)	(0.35)	(0.55)	(0.22)	(0.37)	(0.29)	(0.29)

5.3 European Interdependence

The entropy based measure of financial interdependence in (11) is now applied to identify changes in interdependence amongst euro zone countries, with Germany replacing the U.S. as the benchmark. The empirical results for Europe in Table 6 are generally consistent with those for global interdependence in Table 3. The degree of interdependence $\Psi(\widehat{\Theta}) - \Psi_0$ increases over time from the pre-euro to the euro periods with Belgium, Ireland and Spain joining Austria, France and the Netherlands in achieving greater interdependence with Germany, compared to just France and Germany when using the global measure. As with the global case, European interdependence peaks during the GFC with $\Psi(\widehat{\Theta}) - \Psi_0$ more than doubling from 4.5 to 9.7, with all countries showing greater interdependence with Germany with the exception of Finland. As with the global results, interdependence in Europe falls post GFC with $\Psi(\widehat{\Theta}) - \Psi_0$ moderating to 6.7, which is primarily driven by Greece and to a lesser extent Luxembourg.

Figure 10 shows the time varying refinement of the measure of European financial interdependence with Germany by computing $\Psi(\widehat{\Theta}) - \Psi_0$ over a rolling 3 year window for those countries having data extending over the full sample period. A comparison of the global and European results in Figure 9 and Figure 10 respectively, shows that the time paths are very similar with the main difference being that during the early years of the formation of the euro zone there is statistical evidence of interdependence amongst asset markets within Europe, whereas the results in Figure 9 reveal no evidence with global asset markets.

Table 7 provides a breakdown of the entropy measure of interdependence between euro zone and German equity markets into its components, expressed as a percentage of the total. There are some notable differences between the results in Table 7 and the global interdependence results reported in Table 4. Similar to the global case the covariance channel dominates European equity return interdependence although the proportionate contribution is higher for Europe which ranges between 92% and 98% than for the U.S. where the range is between 80% and 96%. The second most important contributor is again the higher order moment of cokurtosis. Covolatility has a dampening effect on European interdependence throughout the sample period, even during the GFC, while on a global basis covolatility contributed to interdependence during the GFC. This result suggests a fundamental change in interdependence between

Table 6:

European interdependence measure $(\Psi(\widehat{\Theta}) - \Psi_0)$ for the euro zone countries for selected sample periods. Asymptotic standard errors are in parentheses.

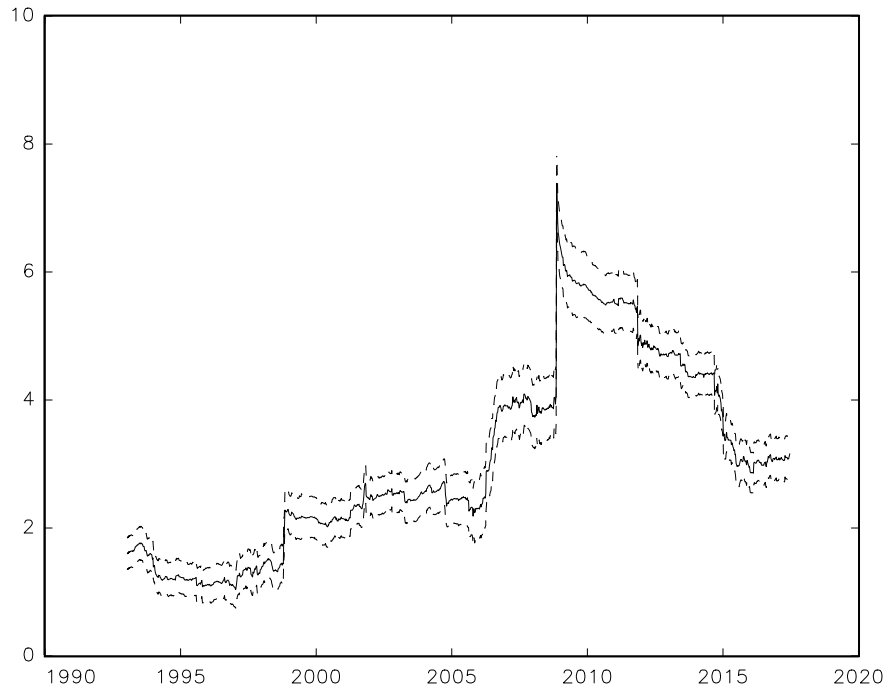| Region /Country	Pre-euro (1990-1998)	Euro (1999-2007)	GFC (2008-2009)	Post-GFC (2010-2017)	Total (1990-2017)
Europe	1.5481	4.5004	9.6611	6.9671	5.8594
	(0.0863)	(0.9307)	(1.0801)	(0.1644)	(0.9338)
Austria	0.3417	0.1457	0.8321	0.6515	0.4868
	(0.2057)	(0.1912)	(0.2996)	(0.2130)	(0.8084)
Belgium	n.a.	0.3750	0.8888	0.8706	0.6762
		(0.2154)	(0.3169)	(0.2650)	(0.2160)
Finland	0.0844	0.1982	0.5882	0.4657	0.2363
	(0.1472)	(0.2151)	(1.0132)	(0.2093)	(0.1941)
France	0.3450	0.7764	1.4777	1.0974	0.7751
	(0.1895)	(0.2315)	(0.3272)	(0.2228)	(0.2986)
Greece	0.0836	0.1466	0.6001	0.2139	0.2011
	(0.1516)	(0.1960)	(0.3588)	(0.2159)	(0.1722)
Italy	n.a.	0.2673	0.5599	0.5930	0.5494
		(0.1964)	(0.3238)	(0.2255)	(0.2440)
Ireland	n.a.	0.7659	1.0124	0.6835	0.7486
		(0.2622)	(0.3686)	(0.2056)	(0.2154)
Luxembourg	n.a.	0.3193	0.5993	0.3643	0.4220
		(0.4054)	(0.3019)	(0.2311)	(0.2903)
Netherlands	0.4666	0.7266	1.0824	0.9689	0.8425
	(0.2062)	(0.2418)	(0.3511)	(0.2185)	(0.1847)
Portugal	n.a.	0.2265	0.8215	0.4591	0.4040
		(0.9237)	(0.2764)	(0.2085)	(0.8037)
Spain	0.2269	0.5159	1.1987	0.5991	0.5174
	(0.1990)	(0.2113)	(0.2998)	(0.2225)	(0.1834)

Figure 10: Time-varying estimates of European interdependence for the euro zone countries using a rolling window of 3 years.

European and global equity markets during the GFC not present amongst European equity markets with Germany.

To investigate the dynamic interactions between European and German equity returns at different points in time, the entropy measure of interdependence is applied for leads and lags of the German returns with those of the euro zone. The results in Table 8 show that European interdependence with the German equity market is only significant on one occasion where the German equity market leads those of Europe during the GFC. For the countries considered individually this is never the case, and it is never the case that interdependence is significant when the German equity market lags those of Europe, indicating the contemporary nature of European interdependence.

6   Conclusions

The examination of the interdependence of asset returns has a long history, spanning a wide range of frameworks and purposes, ranging from models of portfolio optimization, risk management, option pricing, contagion and financial market integration. Higher

Table 7:

Contribution of the weekly European interdependence components for the euro zone, expressed as a percentage, for selected sample periods.

Dependence Measure	Pre-euro (1990-1998)	Euro (1999-2007)	GFC (2008-2009)	Post-GFC (2010-2017)	Total (1990-2017)
Covariance	92.6325	95.4624	98.4260	97.8966	93.5127
Coskewness	0.5090	0.7414	0.4078	0.4047	0.3361
Cokurtosis	15.5626	15.2465	16.4505	25.7070	21.6785
Covolatility	-8.7041	-11.4503	-15.2843	-24.0083	-15.5273

order comoments that contribute to interdependence amongst financial market asset returns are an often neglected aspect of testing and measuring interdependence. Using entropy theory a general procedure is developed to identify changes in asset return interdependence over time. Attention is particularly placed on the changes in comoments such as coskewness, cokurtosis, and covolatility in conjunction with more traditional methods based on covariances. An important advantage of the proposed approach compared to existing entropy based measures of market comovement is that the procedure allows for a natural decomposition of asset returns in terms of the contribution from the higher order comoments. A further advantage is that a new test of independence is developed which arises naturally from the methodological framework.

The proposed approach uses the joint probability distribution to measure the strength of the relationship amongst asset returns and provides an overall measure of the dispersion of this distribution. The chosen distribution is the generalized normal distribution which is a subordinate distribution of the multivariate generalized exponential family of distributions which is a flexible distribution able to capture simultaneously various comoments linking asset returns. To model variations over time in the degree of interdependence amongst asset returns the analysis is performed using a rolling window. A range of simulation experiments based on existing models commonly adopted in empirical finance as data generating processes demonstrate the ability of the approach to capture various forms of asset return interdependence by successfully tracking the comovements in the simulated asset returns over the simulation window.

Applying the entropy measure of interdependence to identifying changes in the comoments of weekly equity returns over time for the initial adopters of the euro

Table 8:

Dynamic weekly European interdependence measure $(\Psi(\widehat{\Theta}) - \Psi_0)$ for the euro zone countries for selected sample periods. Asymptotic standard errors are in parentheses.

Region /Country	Pre-euro (1990-1998)		Euro (1999-2007)		GFC (2008-2009)		Post-GFC (2010-2017)		Total (1990-2017)	
	Lead	Lag	Lead	Lag	Lead	Lag	Lead	Lag	Lead	Lag
Europe	0.17	0.17	0.81	0.89	1.68	1.78	0.39	0.36	1.21	1.12
	(0.74)	(0.86)	(0.55)	(1.82)	(0.87)	(1.72)	(0.37)	(0.42)	(0.76)	(2.18)
Austria	0.04	0.03	0.03	0.03	0.16	0.17	0.03	0.04	0.12	0.12
	(0.32)	(0.28)	(0.21)	(0.30)	(0.54)	(0.63)	(0.32)	(0.41)	(0.52)	(1.08)
Belgium	n.a.	n.a.	0.05	0.05	0.16	0.20	0.05	0.04	0.11	0.12
			(0.20)	(0.27)	(0.42)	(0.75)	(0.35)	(0.37)	(0.48)	(0.58)
Finland	0.04	0.03	0.02	0.02	0.14	0.08	0.07	0.02	0.04	0.02
	(0.19)	(0.33)	(0.21)	(0.19)	(0.49)	(0.42)	(0.39)	(0.23)	(0.34)	(0.14)
France	0.02	0.04	0.04	0.04	0.17	0.14	0.02	0.02	0.10	0.08
	(0.34)	(0.25)	(0.26)	(0.27)	(0.55)	(0.65)	(0.36)	(0.22)	(0.43)	(0.39)
Greece	0.04	0.02	0.05	0.03	0.10	0.21	0.02	0.04	0.03	0.06
	(0.23)	(0.33)	(0.20)	(0.20)	(0.31)	(0.33)	(0.17)	(0.29)	(0.12)	(0.16)
Italy	n.a.	n.a.	0.06	0.06	0.15	0.15	0.03	0.03	0.19	0.12
			(0.24)	(0.28)	(0.47)	(0.45)	(0.32)	(0.44)	(0.63)	(1.10)
Ireland	n.a.	n.a.	0.11	0.09	0.13	0.14	0.04	0.04	0.10	0.10
			(0.29)	(0.26)	(0.54)	(1.03)	(0.38)	(0.29)	(0.32)	(0.28)
Luxembourg	n.a.	n.a.	0.33	0.45	0.20	0.18	0.03	0.03	0.25	0.22
			(0.73)	(1.34)	(0.61)	(0.84)	(0.30)	(0.22)	(0.58)	(0.78)
Netherlands	0.02	0.03	0.05	0.05	0.15	0.19	0.03	0.03	0.11	0.10
	(0.85)	(0.43)	(0.26)	(0.27)	(0.53)	(0.82)	(0.31)	(0.44)	(0.52)	(1.09)
Portugal	n.a.	n.a.	0.04	0.04	0.16	0.17	0.03	0.03	0.08	0.10
			(0.15)	(0.23)	(0.51)	(0.37)	(0.34)	(0.34)	(0.43)	(0.25)
Spain	0.03	0.02	0.04	0.04	0.15	0.14	0.03	0.03	0.07	0.07
	(0.26)	(0.90)	(0.18)	(0.21)	(0.51)	(0.59)	(0.33)	(0.37)	(0.32)	(0.32)

currency using a sample period from the 1990s to early 2017, shows that there is an increase in interdependence in equity returns both within Europe as well as globally, corresponding to the introduction of the euro in 1999. The global financial crisis (GFC) provides a further stimulus to increasing the interdependence amongst European asset returns, before returning to levels experienced just prior to the GFC. The results also show significant evidence of interdependence of equity markets within Europe prior to the introduction of the Euro, but not with global equity markets.

A   Optimality Properties of the Generalized Normal Distribution

The generalized normal distribution has the property that it represents the maximum entropy distribution provided that it satisfies a set of constraints corresponding to the desired moments and comoments that capture interdependence amongst asset returns. Let $f(z_1, z_2)$ be the joint density function where z_1 and z_2 are standardized random variables with zero means and unit variances. The aim is to choose $f(z_1, z_2)$ subject to the following constraints

$$
\begin{aligned}
1 &= \int \int f(z_1, z_2)\, dz_1 dz_2, \\
1 &= \int \int z_1^2 f(z_1, z_2)\, dz_1 dz_2, \quad 1 = \int \int z_2^2 f(z_1, z_2)\, dz_1 dz_2 \\
\sigma_{12} &= \int \int z_1 z_2 f(z_1, z_2)\, dz_1 dz_2, \quad S_{12} = \int \int z_1 z_2^2 f(z_1, z_2)\, dz_1 dz_2 \qquad (22)\\
S_{21} &= \int \int z_1^2 z_2 f(z_1, z_2)\, dz_1 dz_2, \quad K_{13} = \int \int z_1 z_2^3 f(z_1, z_2)\, dz_1 dz_2 \\
K_{31} &= \int \int z_1^3 z_2 f(z_1, z_2)\, dz_1 dz_2, \quad K_{22} = \int \int z_1^2 z_2^2 f(z_1, z_2)\, dz_1 dz_2.
\end{aligned}
$$

The first constraint is the adding up constraint ensuring that $f(z_1, z_2)$ is a proper bivariate density function. The next two constraints are the normalizing conditions corresponding to unit variances. The next set of constraints are respectively the covariance, the coskewness terms S_{12} and S_{21}, the cokurtosis terms K_{13} and K_{31}, and the covolatility term K_{22}.

To choose a density function $f(z_1, z_2)$ that maximizes entropy subject to the constraints in (22) define the constrained objective function

$$
\begin{aligned}
J &= \int \int (\log f(z_1, z_2)) f(z_1, z_2)\, dz_1 dz_2 - \lambda_0 \left(1 - \int \int f(z_1, z_2)\, dz_1 dz_2 \right) \\
&\quad - \lambda_1 \left(1 - \int \int z_1^2 f(z_1, z_2)\, dz_1 dz_2 \right) - \lambda_2 \left(1 - \int \int z_2^2 f(z_1, z_2)\, dz_1 dz_2 \right) \\
&\quad - \lambda_3 \left(\sigma_{12} - \int \int z_1 z_2 f(z_1, z_2)\, dz_1 dz_2 \right) - \lambda_4 \left(S_{12} - \int \int z_1 z_2^2 f(z_1, z_2)\, dz_1 dz_2 \right) \\
&\quad - \lambda_5 \left(S_{21} - \int \int z_1^2 z_2 f(z_1, z_2)\, dz_1 dz_2 \right) - \lambda_6 \left(K_{13} - \int \int z_1 z_2^3 f(z_1, z_2)\, dz_1 dz_2 \right) \\
&\quad - \lambda_7 \left(K_{31} - \int \int z_1^3 z_2 f(z_1, z_2)\, dz_1 dz_2 \right) - \lambda_8 \left(K_{22} - \int \int z_1^2 z_2^2 f(z_1, z_2)\, dz_1 dz_2, \right),
\end{aligned}
$$

$$(23)$$

where the first term defines the entropy of a distribution and the $\lambda_i s$ are the Lagrange multiplers. Using variational calculus, a small variation δf about f produces a variation δJ about J of

$$
\begin{aligned}
\delta J \;=\;& \int\int \delta f\,(z_1, z_2)\, dz_1 dz_2 + \int\int (\log f\,(z_1, z_2))\,\delta f\,(z_1, z_2)\, dz_1 dz_2 \\
&+\lambda_0 \int\int \delta f\,(z_1, z_2)\, dz_1 dz_2 + \lambda_1 \int\int z_1^2 \delta f\,(z_1, z_2)\, dz_1 dz_2 \\
&+\lambda_2 \int\int z_2^2 \delta f\,(z_1, z_2)\, dz_1 dz_2 + \lambda_3 \int\int z_1 z_2 \delta f\,(z_1, z_2)\, dz_1 dz_2 \\
&+\lambda_4 \int\int z_1 z_2^2 \delta f\,(z_1, z_2)\, dz_1 dz_2 + \lambda_5 \int\int z_1^2 z_2 \delta f\,(z_1, z_2)\, dz_1 dz_2 \\
&+\lambda_6 \int\int z_1 z_2^3 \delta f\,(z_1, z_2)\, dz_1 dz_2 + \lambda_7 \int\int z_1^3 z_2 \delta f\,(z_1, z_2)\, dz_1 dz_2 \\
&+\lambda_8 \int\int z_1^2 z_2^2 \delta f\,(z_1, z_2)\, dz_1 dz_2.
\end{aligned}
$$

Collecting terms gives

$$
\begin{aligned}
\delta J \;=\;& \int\int \delta f\,(z_1, z_2)\,\big\{1 + \log f\,(z_1, z_2) + \lambda_0 + \lambda_1 z_1^2 + \lambda_2 z_2^2 + \lambda_3 z_1 z_2 \\
&+\lambda_4 z_1 z_2^2 + \lambda_5 z_1^2 z_2 + \lambda_6 z_1 z_2^3 + \lambda_7 z_1^3 z_2 + \lambda_8 z_1^2 z_2^2 \big\}\, dz_1 dz_2. \qquad (24)
\end{aligned}
$$

A maximum of J in (23) is achieved by setting $\delta J = 0$ in (24) which requires that the term in parentheses in (24) is also zero

$$
1+\log f\,(z_1, z_2)+\lambda_0+\lambda_1 z_1^2+\lambda_2 z_2^2+\lambda_3 z_1 z_2+\lambda_4 z_1 z_2^2+\lambda_5 z_1^2 z_2+\lambda_6 z_1 z_2^3+\lambda_7 z_1^3 z_2+\lambda_8 z_1^2 z_2^2 = 0.
$$

Rearranging this expression by solving for $f\,(z_1, z_2)$ yields the generalized normal distribution

$$
\begin{aligned}
f\,(z_1, z_2) \;=\;& \exp\big(-1 - \lambda_0 - \lambda_1 z_1^2 - \lambda_2 z_2^2 - \lambda_3 z_1 z_2 - \lambda_4 z_1 z_2^2 - \lambda_5 z_1^2 z_2 - \lambda_6 z_1 z_2^3 \\
&-\lambda_7 z_1^3 z_2 - \lambda_8 z_1^2 z_2^2 \big),
\end{aligned}
$$

or

$$
\begin{aligned}
\;=\;& \exp\Big(-\frac{1}{2}\Big(\frac{1}{1-\rho^2}\Big)(z_1^2 + z_2^2 - 2\rho z_{12}) + \theta_1 z_1 z_2^2 + \theta_2 z_1^2 z_2 + \theta_3 z_1 z_2^3, \\
&+\theta_4 z_1^3 z_2 + \theta_5 z_1^2 z_2^2 - \eta \Big),
\end{aligned}
$$

where the normalizing constant is $\eta = 1+\lambda_0$, $\lambda_1 = \lambda_2 = 0.5/\,(1 - \rho^2)$, $\lambda_3 = -\rho/\,(1 - \rho^2)$, and $\theta_i = -\lambda_{i+3}$, $i = 1, 2, 3, 4, 5$.

B Additional Simulation Properties

This appendix reports some additional simulation results of the properties of the entropy interdependence measure based on the Bekaert-Harvey and Solnik-Roulet models presented in Sections 4.1 and 4.2 respectively.

B.1 Bekaert-Harvey Model

The results of an alternative simulation of the Bekaert-Harvey model to the one given in Figure 5 are presented in Figure 11. The data generating process is based on (21) for a sample of size $T = 2000$, except that the parameters used to generate the time-varying integration parameter ϕ_t, differ. The two graphs in the first column of Figure 11 give the simulated world returns (r_{wt}) and the country returns (r_{it}). The parameter ϕ_t shows that there is near full global integration at the start of the period with conditions changing to market segmentation roughly two thirds the way through the sample. This is immediately followed by a period of increasing global integration eventually stabilizing on a level of global integration less than that achieved at the start of the sample period.

The bottom panel of the second column shows that the entropy interdependence measure Ψ_t, is able to track the latent global integration measure. As with the integration measure ϕ_t, Ψ_t also starts off at a high value consistent with r_{wt} and r_{it} being interdependent. The interdependence measure then begins to fall as the relative importance of local factors start to increase. This is followed by a period where the entropy measure shows increasing interdependence between the two returns, albeit with some large swings, eventually settling upon a level of interdependence that is also less than the level that occurred at the start of the sample. The correlation between ϕ_t and Ψ_t for this simulation run is 0.70.

B.2 Solnik-Roulet Model

Figure 11 provides the results of an alternative simulation of the Solnik-Roulet model to the one given in Figure 7 where the number of countries in the sample is doubled to $N = 40$. The sample size is still held at $T = 2000$. The top graph in Figure 12 gives the Solnik-Roulet measure σ_t, which is based on the cross-sectional standard deviation of the $N = 40$ returns at each point in time. The degree of interdependence is relatively
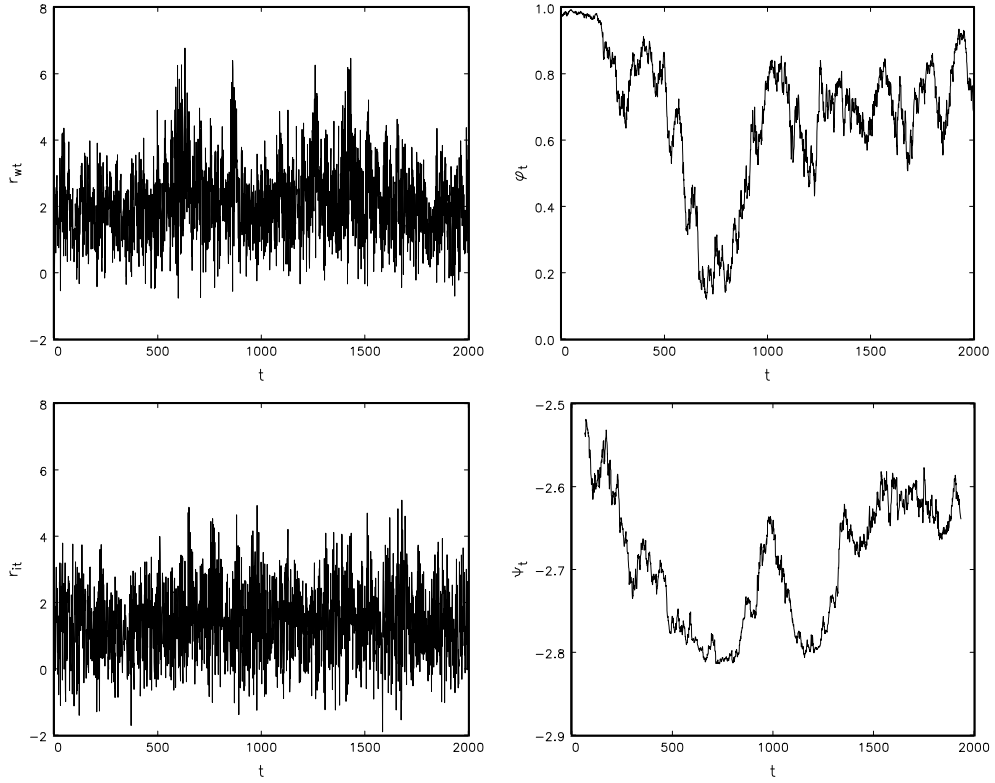
Figure 11: Altenative simulation results of the Bekaert-Harvey model in (21) based on a sample size of $T = 2000$. The first column gives the simulated returns for the world (r_{wt}) and the country (r_{it}). The top panel of the second column gives the measure of financial integration (ϕ_t) with $\phi_t = 1$ representing global market integration and $\phi_t = 0$ representing market segmentation. The bottom panel of the second column gives the interdependence measure Ψ_t applied to the simulated returns with a data window of 130 observations.
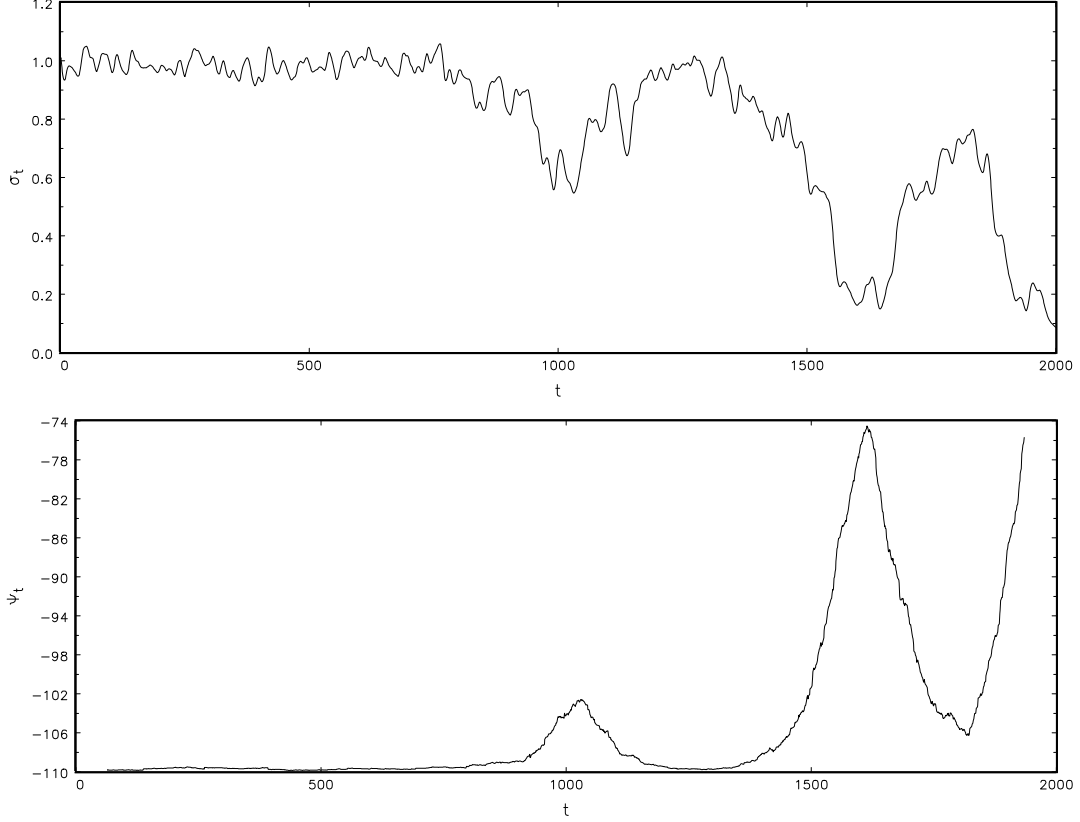
Figure 12: Simulation results of the Solnik-Roulet model based on a sample size of $T = 2000$ with $N = 40$ asset markets. The first panel gives the measure of financial interdependence (σ_t) with falls in σ_t representing increasing financial interdependence and increases in σ_t representing movements towards independence. The second panel is the interdependence measure Ψ_t applied to the simulated country and global returns with a data window of 130 observations.

stable for most of the first half of the sample. In the second half of the sample, σ_t oscillates around a negative trend as asset returns become more interdependent. The entropy interdependence measure Ψ_t given in the bottom panel of Figure 12 mirrors the movements in σ_t as it is flat during the first half of the sample, while in the second half it has a positive trend albeit with large oscillations reflecting that there is increasing interdependence over this period. The correlation between σ_t and Ψ_t is 0.94.

C   Sensitivity Results

Figures 13 and 14 respectively give the global and European estimates of time-varying interdependence estimates for alternative windows. The window widths are 2-years as
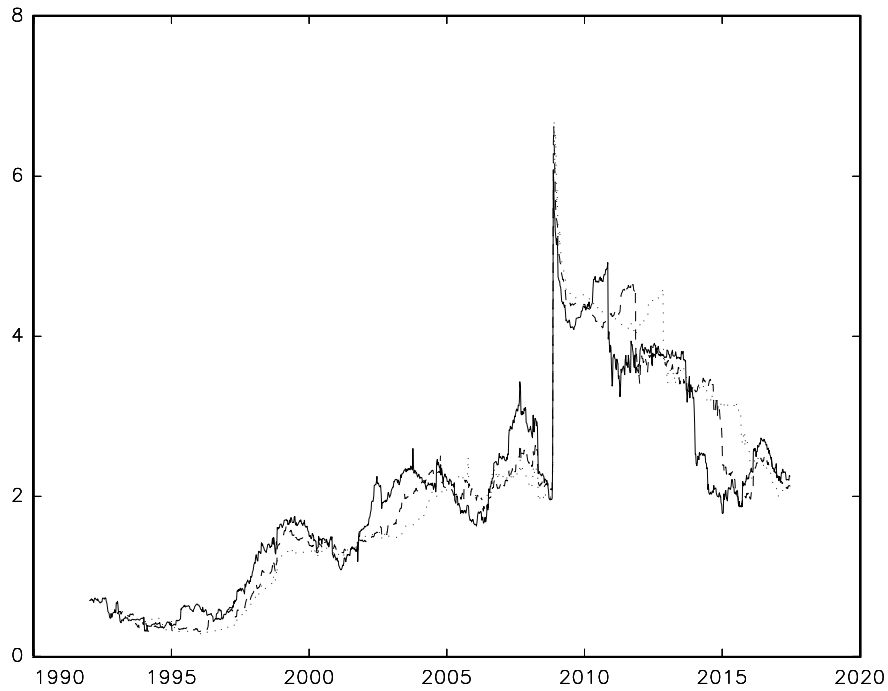
41

Figure 13: Time-varying global entropy interdependence estimates for alternative windows: 2-year (continuous), 3-year (short dashes), 4-year (dotted).

given by the continuous line 3-years as given by the short dashes, and 4-years as given by the dotted line. All three sets of estimates in each graph track the overall changes in interdependence over time. As to be expected wider window widths tend to yield smoother Ψ_t estimates.
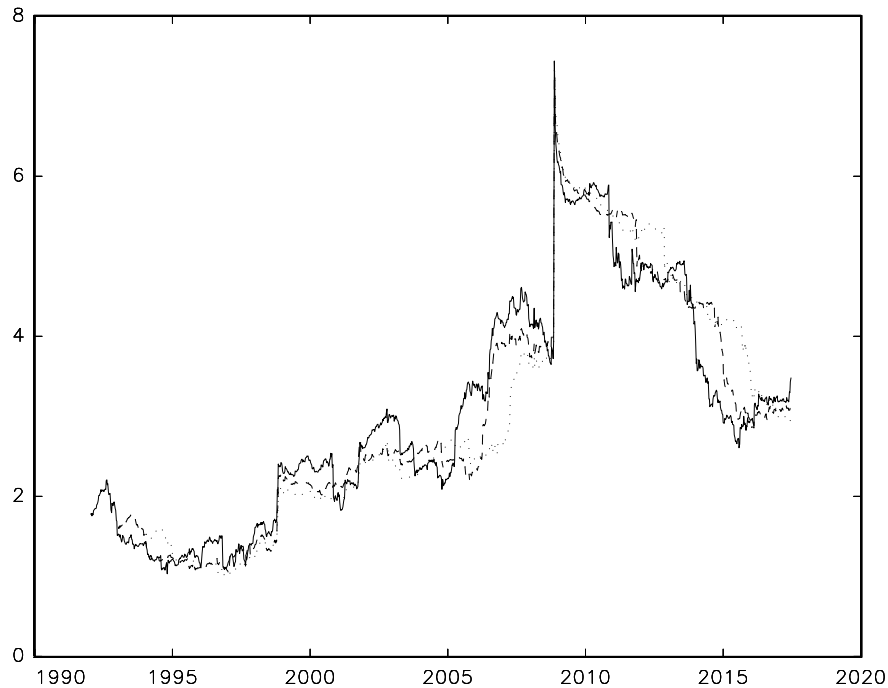
Figure 14: Time-varying European entropy interdependence estimates for alternative windows: 2-year (continuous), 3-year (short dashes), 4-year (dotted).

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